Week 16

Multiple Equation Models

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Multiple equation models

- Also known as simultaneous equation models or systems of equations.
- Why on earth would we ever want to use these models?
- Theory!
- We have reason to believe that our data violate the assumptions underlying the other models we have seen this semester.

Multiple equation models

- The most important assumption for the purposes of this class is that the effects that we are modeling are unidirectional—our *x*'s affect our *y*'s, and our *y*'s have **no** effect on our *x*'s.

- However, the social world that we as political scientists are interested in is rarely that simple.

- We have already begun relaxing the assumption that one equation can adequately approximate the phenomena we are interested in.

- Therefore, we have already become familiar with multiple equation models including the ZIP, ZINB, Heckman, and other selection models.

- Therefore, this class is but another effort at relaxing restrictive assumptions we have made in the past.

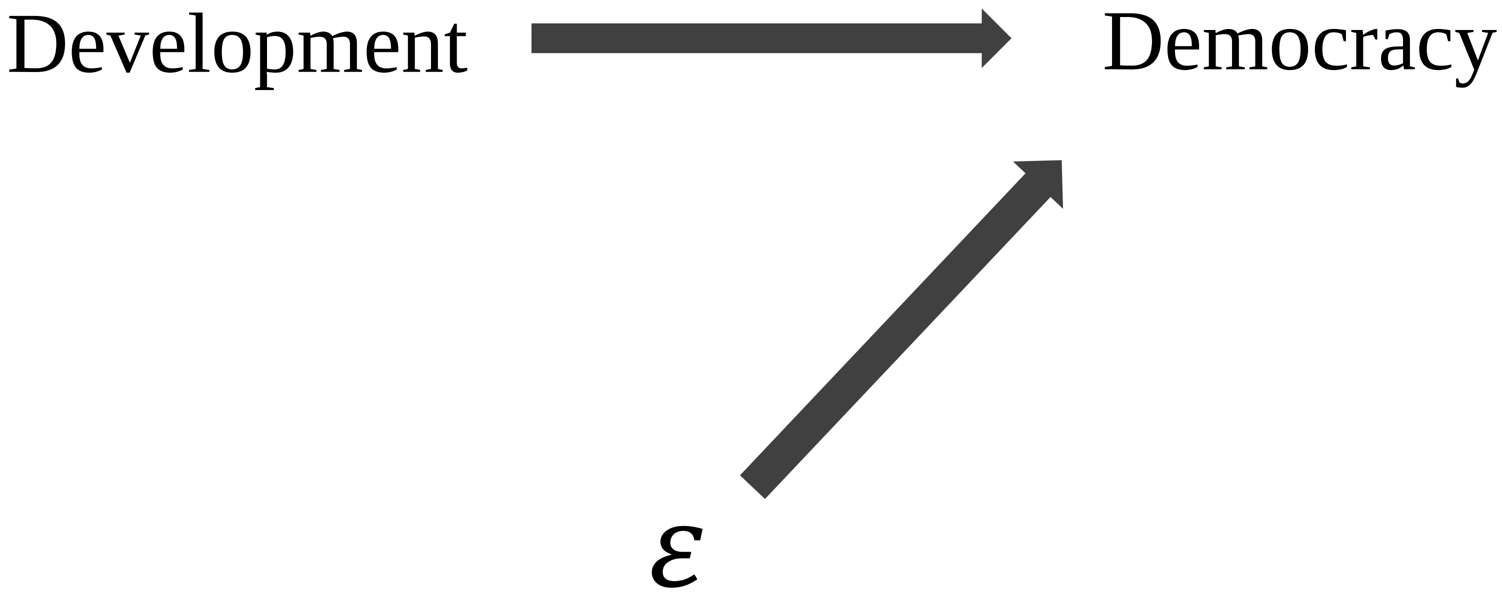
- One of the most important of these assumptions (dating back to 6002) is that our *x*'s are unrelated to our error term, ε.

- If a regressor (let's call it Y_2) is associated with our disturbance term, ε_1, then when our disturbances increase, Y_1 increases.

- This means (in an additional complication we will look at in a few minutes) that Y_2 would be affected if Y_1 was a predictor of Y_2.

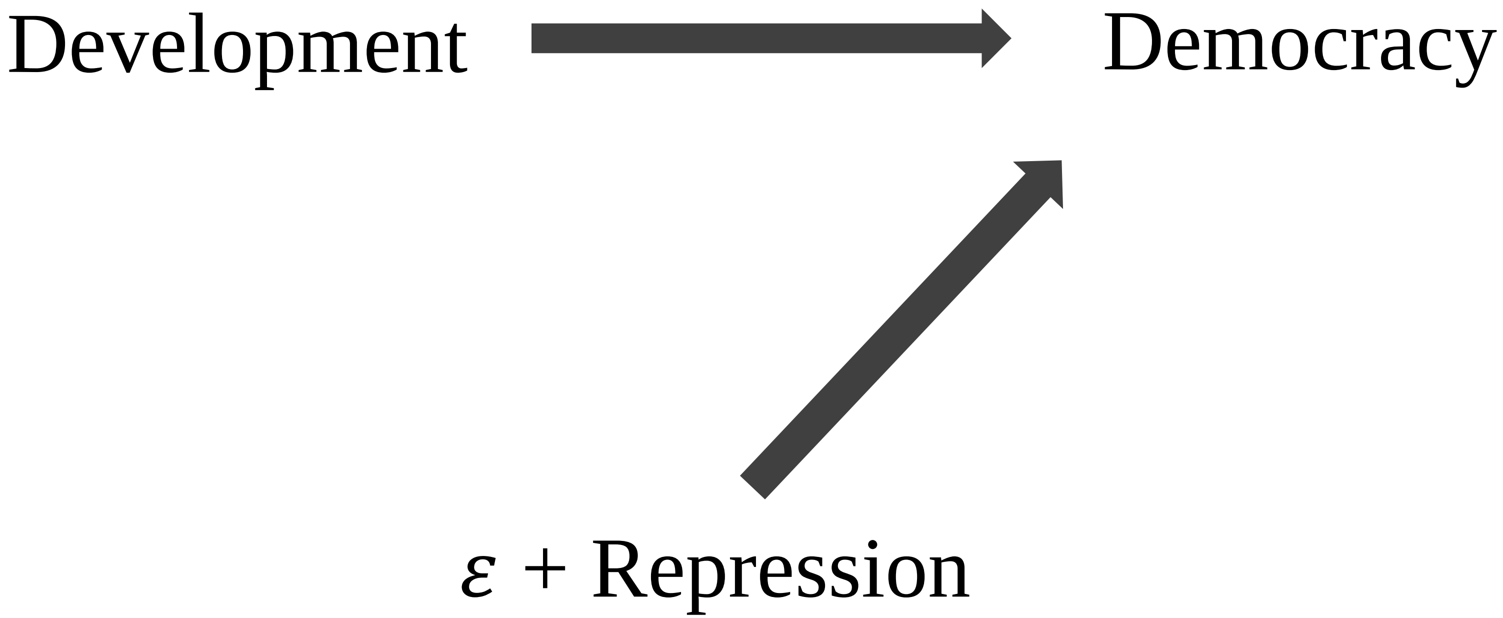
- For example, think about economic development and democracy.

- There is a large literature suggesting that economically developed states are more likely to be democratic.


Development
Democracy
ε

- In this example, ε includes everything else besides economic development that can cause democracy.

- One such factor that can effect democracy could be the government's decision to repress its citizens.


Development
Democracy
ε + Repression

- But repression can also affect corporations and citizens from wanting to invest in their future.

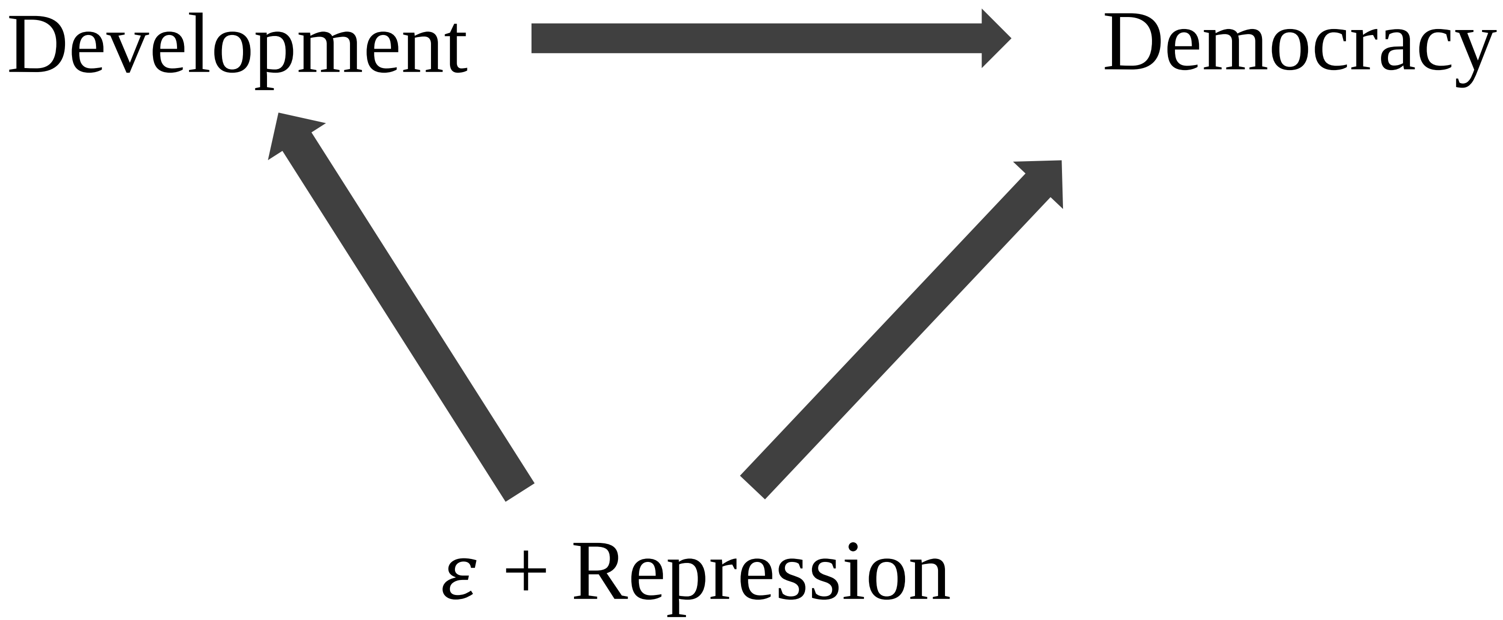
- People might be more worried about whether security forces are going to throw them out of a helicopter than on starting a new business.

- Another popular example is the relationship between party affiliation and candidate evaluations.

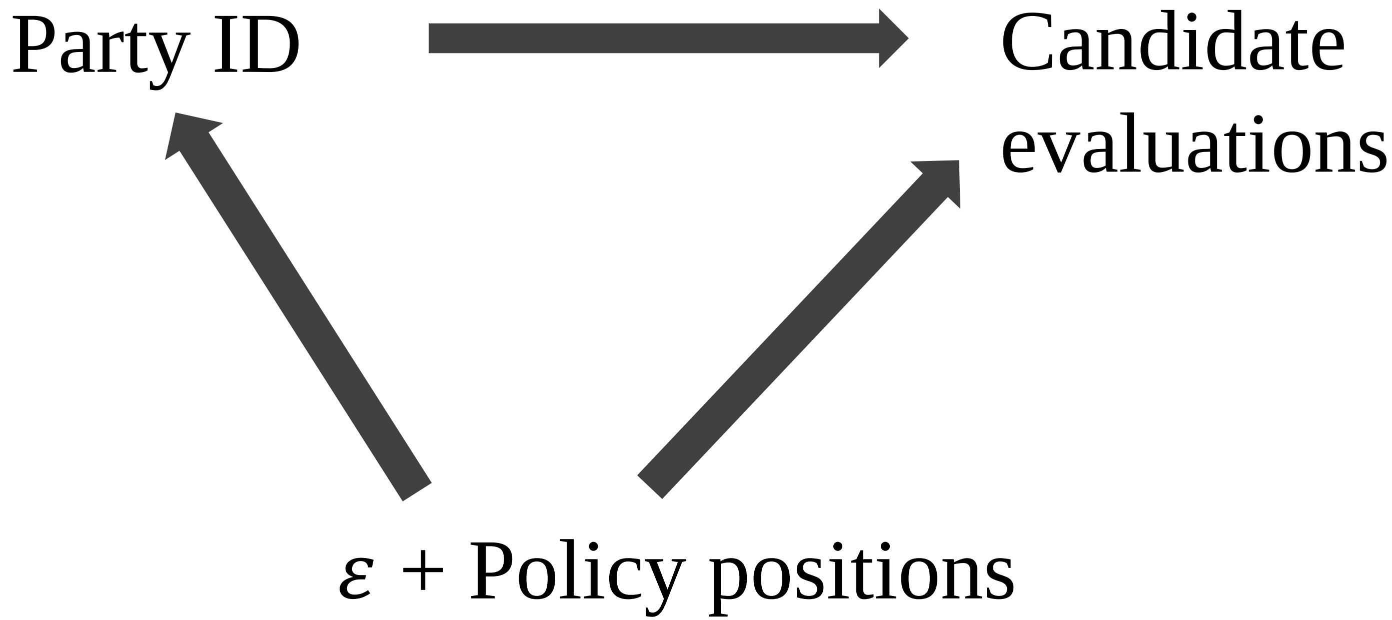
- The positions that candidates have can affect what party they join *as well as* how they are evaluated.

- You can also model these types of theoretical relationships.

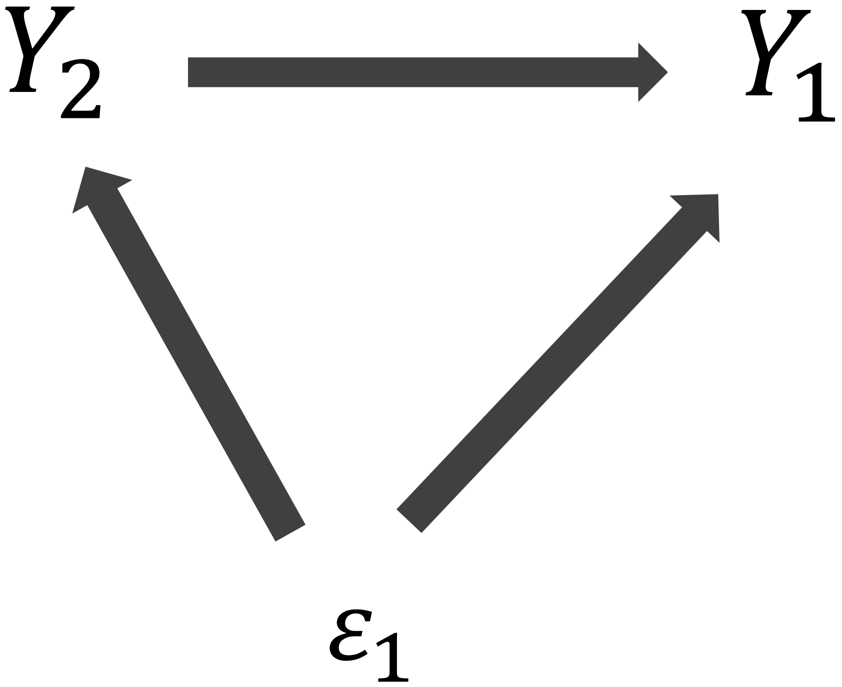
The path diagram should look more like this



Or this



In notation



- This means that Y_2, economic development, is an **endogenous** regressor, because it arises in a system where it is related to ε.

- A variable that affects democracy (say distance from Geneva) but does not affect economic development would be considered **exogenous**.

- If we just run a single equation predicting democracy without taking into account what we know about the reciprocal nature of the relationship between democracy and development, then our OLS estimators would be **inconsistent**.

- Remember, an estimate is considered consistent if as our sample approaches the size of the population our parameter value approximates the population's value.

- Therefore in order to get consistent and efficient estimators, we need to take into account what we theoretically know about the world and model this endogeneity.

- This simple example and path diagram hints that there are a large number of potential models that can be adjusted or tweaked to fit the **theoretical** model (path diagram) that we think explains the world.

- We will explore several of the most straightforward and popular models today.
  - First, the adjustments we need to make about the distribution of our errors given our theoretical model.
  - And then explicitly model these endogenous effects.

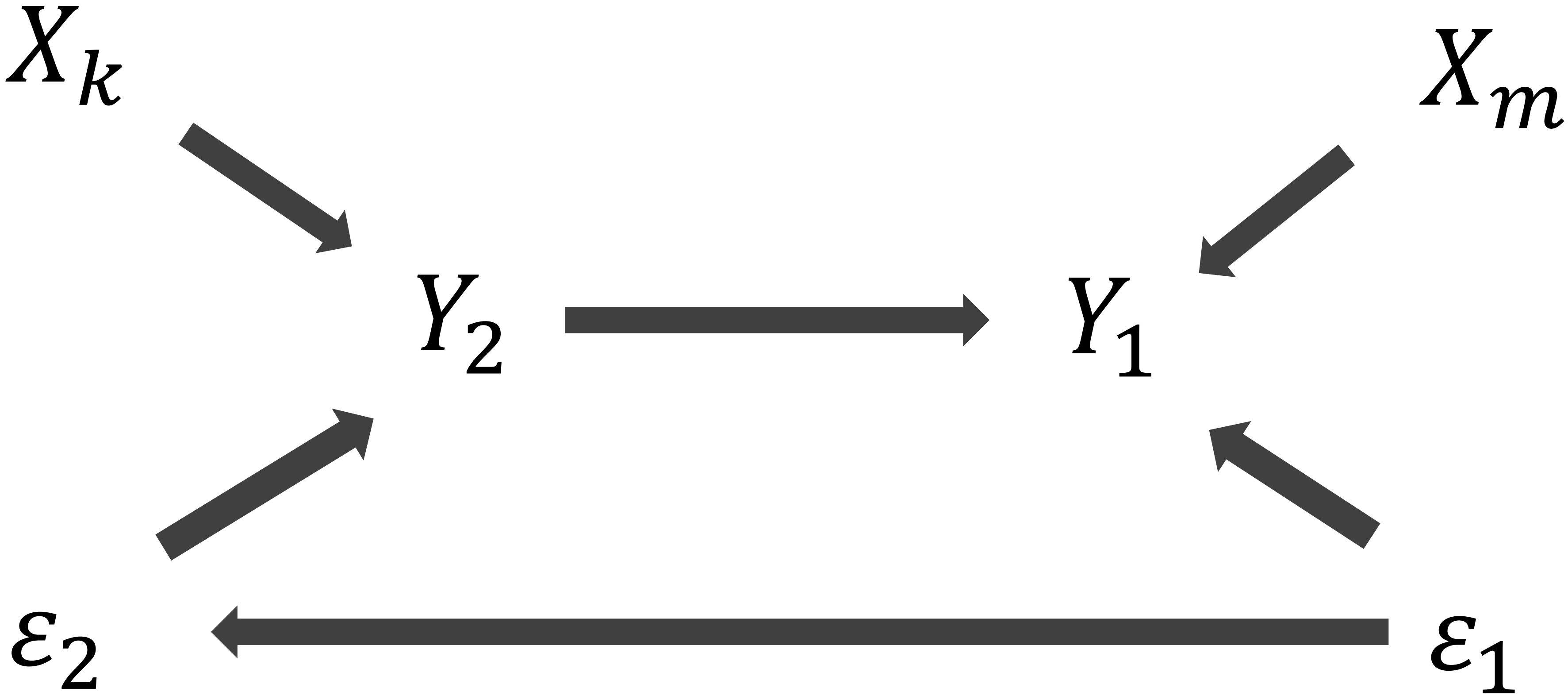
- A basic structure of equations can be written as follows:

$$Y_1 = \beta_{01} + \boxed{\beta_{11} Y_2} + \boldsymbol{\beta_m X_m} + \varepsilon_1$$

$$Y_2 = \beta_{02} + \boldsymbol{\beta_k X_k} + \varepsilon_2$$

- Where $\boldsymbol{X_m}$ represents a vector of variables that affect Y_1, and $\boldsymbol{X_k}$ represents a vector of variables that affect Y_2.

- And *m* and *k* are >0


X_k
X_m
Y_2
Y_1
ε_2
ε_1

- To obtain a consistent parameter estimate, we assume that ε_1 is uncorrelated with $\boldsymbol{X_m}$ but are correlated with Y_2.

- In order to estimate the model we also need at least k variables that satisfy the assumption that $E(\varepsilon_1 | \boldsymbol{X_k}) = 0$.

- These k variables therefore have to provide some information about Y_2 but have no effect on ε_1.

- This model is relatively simple to estimate because all the arrows point in one direction (uni-directional).
- The models are hierarchical.
- This type of model is called **recursive**.

- However, there are many instances where these assumptions are non-realistic.

- Going back to my development and democracy example, it has been argued that democracies are more likely to attract investment and development because they are perceived to be more stable or have clearer rules and institutional decision-making.

- Therefore, development affects democracy at the same time that democracy affects development.

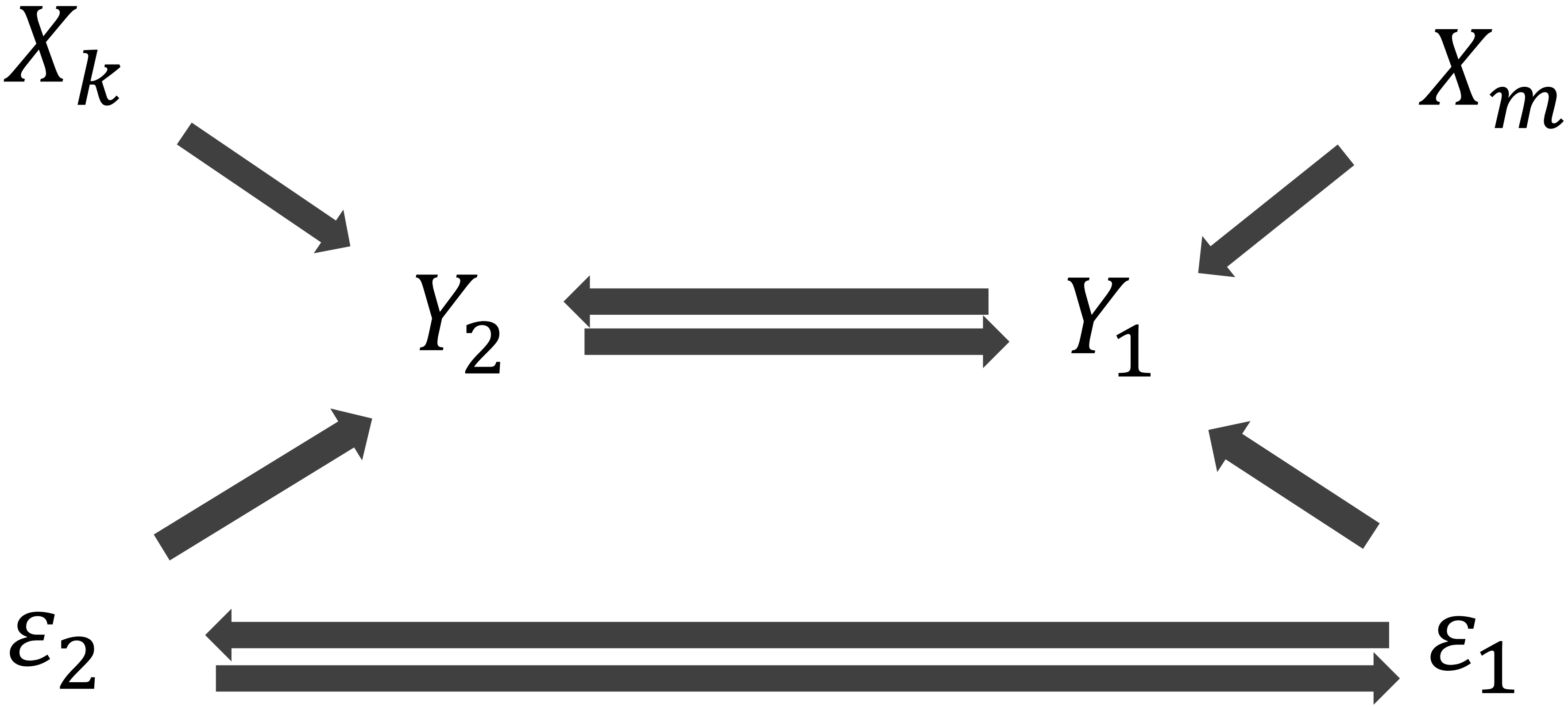
Non-recursive models

- We could therefore write a more complicated **non-recursive** system of equations:

$$Y_1 = \beta_{01} + \boxed{\beta_{11} Y_2} + \boldsymbol{\beta_m X_m} + \varepsilon_1$$

$$Y_2 = \beta_{02} + \boxed{\beta_{12} Y_1} + \boldsymbol{\beta_k X_k} + \varepsilon_2$$

Here, arrows go both directions.



Limited information models

- If the system is recursive, then you can run the models for Y_1 and Y_2 independently.
- This is considered a *limited information approach* because we are not using all the information we have on what we know about the world.

The identification problem

- As Greene (2008: 361-2) discusses, if there exists more than one theory that can lead to the same observed data (observational equivalence), then the model structure is **unidentified**.

- Simultaneous equation models are considered one of three types:
  - Under-identified
  - Identified
  - Over-identified

Observational equivalence

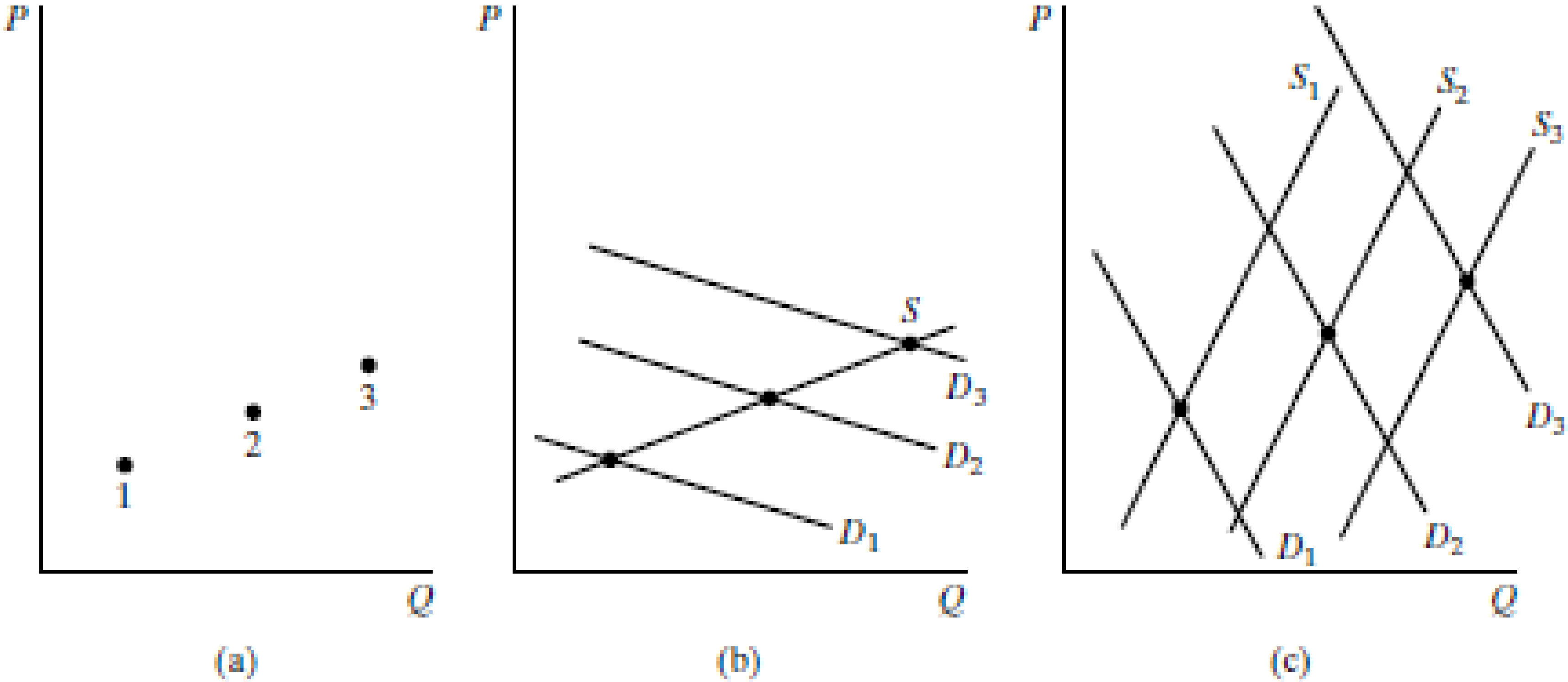


FIGURE 15.1 Market Equilibria.

Source: Greene (2005)

Order condition for identification

- A necessary (but not sufficient) condition for identification is that:

- "In a model of M simultaneous equations in order for an equation to be identified, it must exclude at least $M - 1$ variables (endogenous as well as predetermined [exogenous]) appearing in the model. If it excludes exactly $M -1$ variables, the equation is just identified. If it excludes more than $M -1$ variables, it is overidentified,"

-Gujarati (2003: 748)

It is easiest to use a table of coefficients

	Coefficients				
	Intercept	Y1	Y2	X1	X2
Y1	B01	1	B12	B13	0
Y2	B02	0	1	0	B23

Rank Condition for Identification

- The order condition, you will notice, is necessary, but it is not sufficient to identify a model.
- There will be at least one solution, but there could be more than one.
- We need another condition that is sufficient for uniqueness.

Rank Condition for Identification

- "In a model containing M equations in M endogenous variables, an equation is identified if and only if at least one nonzero determinant of order (M - *1*)(M - *1*) can be constructed from the coefficients of the variables (both endogenous and predetermined) excluded from that particular equation but included in the other equations of the model,"

Gujarati (2003: 750)

- The rank order condition involves establishing matrices of the excluded variables of a particular equation that are included in another equation.
- This would take an additional class to explain to my (and probably your satisfaction).
- Suffice it to say that there is a means of establishing the rank order condition manually, and Stata will reject your model if it is not at least identified.
- Let’s move on to estimating models with endogenous regressors.

Two-Stage Lease Squares (2SLS)

- Developed independently by Theil (1953) and Basmann (1957).
- For example, take our **non-recursive model** above where Y_1 and Y_2 (democracy and development, say) were functions of each other.
- **Stage 1**: Regress Y_1 on all exogenous variables in the system. This gives you the $\widehat{Y_1}$ and $\hat{\varepsilon}$.
- **Stage 2**: Plug in the predicted Y_1 ($\widehat{Y_1} + \hat{\varepsilon}$). This estimates an error term that includes the estimated error $\widehat{\varepsilon_1}$ and ε_2.
- Substantively what this does is asymptotically "purify" Y_1 from the effect of ε_2.
- This enables us to get **consistent** estimates as our sample size increases towards the population size.

Three-stage Lease Squares (3SLS)

- **Stage 1**: Same as for 2SLS, but for all equations.

- **Stage 2**: Estimate covariance matrix of disturbances from the Stage 1 estimates of all the endogenous regressors models

- **Stage 3**: Using the Stage 2 matrix plug in the instrumented variable values rather than the endogenous variables.

Three-stage Lease Squares (3SLS)

- Basically, the difference between 2SLS and 3SLS is that 3SLS runs models for **all** endogenous variables not just the first.
- Using 3SLS with equations that are identified (not over identified) will lead to identical estimates to 2SLS.
- However, 3SLS is more susceptible to model misspecification (because it depends on having the correct matrix of disturbances).
- A benefit is that there are more postestimation tests available (`test` or `testnl`).

- Let’s try some examples…

- Namely, the interrelationship between democracy and development using Fearon and Laitin’s (2003) data.

Polity without controlling for endogeneity

```
. reg polity2 gdpenl colbrit mtnest Oil ef warl, robust cluster(ccode)

Linear regression                                      Number of obs =    6243
                                                       F(  6,   152) =   11.26
                                                       Prob > F      =  0.0000
                                                       R-squared     =  0.2254
                                                       Root MSE      =  6.6845

                                (Std. Err. adjusted for 153 clusters in ccode)
------------------------------------------------------------------------------
             |               Robust
     polity2 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      gdpenl |   .6320265   .2112486     2.99   0.003     .2146639    1.049389
     colbrit |   2.192107    .976455     2.24   0.026     .2629305    4.121283
      mtnest |   .0065496   .0191193     0.34   0.732    -.0312243    .0443235
         Oil |  -5.102077   1.247602    -4.09   0.000    -7.566957   -2.637198
          ef |  -4.433521   1.916182    -2.31   0.022     -8.21931   -.6477328
        warl |   1.556295   .8451118     1.84   0.067     -.113387    3.225977
       _cons |  -.8329061   1.597662    -0.52   0.603    -3.989398    2.323585
------------------------------------------------------------------------------

. est store P_OLS
```

With 2SLS: first stage

```
 ivregress 2sls polity2 colbrit mtnest Oil ef warl (gdpenl = Oil year muslim relfrac), ///
>  first robust cluster(ccode)

First-stage regressions
-----------------------
                                                   Number of obs   =       6243
                                                   N. of clusters  =        153
                                                   F(   8,   6234) =      13.32
                                                   Prob > F        =     0.0000
                                                   R-squared       =     0.2034
                                                   Adj R-squared   =     0.2024
                                                   Root MSE        =     3.9505
------------------------------------------------------------------------------
             |               Robust
      gdpenl |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     colbrit |   1.329785   .7243687     1.84   0.066    -.0902269    2.749798
      mtnest |  -.0069368   .0144117    -0.48   0.630    -.0351887     .021315
         Oil |    3.25208   .9360339     3.47   0.001     1.417131    5.087029
          ef |   -4.67675   .9701836    -4.82   0.000    -6.578644   -2.774856
        warl |  -1.919467   .5477171    -3.50   0.000    -2.993181    -.845753
        year |   .0646421   .0094885     6.81   0.000     .0460414    .0832428
      muslim |  -.0111938   .0070446    -1.59   0.112    -.0250037    .0026161
     relfrac |   2.248686   1.475143     1.52   0.127    -.6431015    5.140474
       _cons |  -122.8343   18.38209    -6.68   0.000    -158.8695   -86.79907
------------------------------------------------------------------------------
```

Second stage

```
Instrumental variables (2SLS) regression               Number of obs =    6243
                                                       Wald chi2(6)  =   66.39
                                                       Prob > chi2   =  0.0000
                                                       R-squared     =  0.0445
                                                       Root MSE      =  7.4198

                                (Std. Err. adjusted for 153 clusters in ccode)
------------------------------------------------------------------------------
             |               Robust
     polity2 |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      gdpenl |   1.417127   .2935135     4.83   0.000     .8418509    1.992403
     colbrit |    .920523   1.327838     0.69   0.488    -1.681991    3.523037
     mtnest  |    .015147   .0163096     0.93   0.353    -.0168192    .0471133
         Oil |  -7.323945   2.100681    -3.49   0.000     -11.4412   -3.206686
          ef |  -1.125375    1.70013    -0.66   0.508    -4.457568    2.206818
        warl |   2.805521   .9799865     2.86   0.004     .8847828    4.726259
       _cons |  -4.921945   1.717969    -2.86   0.004    -8.289102   -1.554787
------------------------------------------------------------------------------
Instrumented:  gdpenl
Instruments:   colbrit mtnest Oil ef warl year muslim relfrac

. est store P_2SLS
```

Notice the difference in GDP's Beta

```
. est table P_OLS P_2SLS, b(%9.5f) se

--------------------------------------
    Variable |   P_OLS       P_2SLS
-------------+------------------------
      gdpenl |   0.63203      1.41713
             |   0.21125      0.29351
     colbrit |   2.19211      0.92052
             |   0.97646      1.32784
      mtnest |   0.00655      0.01515
             |   0.01912      0.01631
         Oil |  -5.10208     -7.32394
             |   1.24760      2.10068
          ef |  -4.43352     -1.12538
             |   1.91618      1.70013
        warl |   1.55630      2.80552
             |   0.84511      0.97999
       _cons |  -0.83291     -4.92194
             |   1.59766      1.71797
--------------------------------------
                           legend: b/se
```

Testing for endogeneity

- If the variables that we think are endogenous are really exogenous, then we would gain efficiency by just using OLS.
- The Hausman test provides one popular means of gauging endogeneity by comparing the coefficients of the endogenous regressors.

Testing for endogeneity

- For one (potentially) endogenous variable, the Hausman (1978) test statistic is:

$$T_H = \frac{(\widehat{\beta_{IV}} - \hat{\beta}_{OLS})^2}{\hat{V}(\hat{\beta}_{IV} - \hat{\beta}_{OLS})}$$

- This test statistic is distributed chi-squared with 1 degree of freedom.

Testing for endogeneity

```
.    estat endogenous, forcenonrobust

  Tests of endogeneity
  Ho: variables are exogenous

  Durbin (score) chi2(1)            =   123.603   (p = 0.0000)
  Wu-Hausman F(1,6235)              =   125.938   (p = 0.0000)
```

- Stata also reports a Durbin test statistic, which assumes exogeneity and tests for endogeneity (the opposite of the Hausman).

First stage diagnostics

```
.   estat firststage

  First-stage regression summary statistics
  --------------------------------------------------------------------------
               |            Adjusted      Partial
      Variable |   R-sq.       R-sq.        R-sq.     F(3,6234)   Prob > F
  -------------+------------------------------------------------------------
        gdpenl |  0.2034      0.2024       0.0782       176.221    0.0000
  --------------------------------------------------------------------------


  Minimum eigenvalue statistic = 176.221

  Critical Values                      # of endogenous regressors:    1
  Ho: Instruments are weak             # of excluded instruments:     3
  ---------------------------------------------------------------------
                                     |    5%      10%      20%      30%
  2SLS relative bias                 |  13.91     9.08     6.46     5.39
  -----------------------------------+---------------------------------
                                     |   10%      15%      20%      25%
  2SLS Size of nominal 5% Wald test  |  22.30    12.83     9.54     7.80
  LIML Size of nominal 5% Wald test  |   6.46     4.36     3.69     3.32
  ---------------------------------------------------------------------
```

First stage diagnostics

- The partial *R-squared* is the variance that is explained by the instruments after controlling for the endogeneity.
- The *F* statistic is for the joint significance for the instruments.
- There is a rule of thumb that suggests you have strong instruments if the F statistic is greater than 10.

What happens if we misspecify a 2SLS?

```
. ivregress 2sls polity2 Oil (gdpenl = Oil), ///
first robust cluster(ccode)
```

equation not identified; must have at least as many instruments not in the regression as there are instrumented variables

```
r(481);
```

- What happens if instead of some nice continuous variables, we have at least one dichotomous endogenous variable?

- As we have seen numerous times before, if we use least-squares we are going to get heteroskedastic residuals.

- You could ignore them…
- But we know better than to do that because it would lead to biased and inconsistent estimates.

- Let's try dichotomizing Polity into a dummy variable called dichotomous, which equals 1 if Polity>5.

```
. est table P_OLS P_2SLS dich, b(%9.5f) se
```

Variable	P_OLS	P_2SLS	dich
gdpenl	**0.63203**	**1.41713**	**0.08864**
	0.02058	0.08169	0.01773
colbrit	2.19211	0.92052	0.06584
	0.19898	0.25479	0.07956
mtnest	0.00655	0.01515	0.00012
	0.00414	0.00467	0.00089
Oil	-5.10208	-7.32394	-0.39356
	0.26408	0.36768	0.11452
ef	-4.43352	-1.12538	-0.11566
	0.34296	0.50411	0.10113
warl	1.55630	2.80552	0.13521
	0.25690	0.31127	0.06864
_cons	-0.83291	-4.92194	0.08405
	0.21460	0.47286	0.10240

legend: b/se

- In addition to running `ivregress` using 2SLS , you can also specify LIML or GMM.

- These are different means of specifying the βs, and are outside the scope of what I am trying to cover today.

- GMM is a popular alternative to OLS that estimates a parameter by substituting a population parameter (say μ) with its sample equivalent.

$$E(y - \mu) = 0$$

$$\hat{\mu} = \frac{1}{N}\sum_{i=1}^{N} y_i$$

Limited Information Maximum Likelihood (LIML)

- Also referred to as the least-variance ratio.
- More computationally intensive (you do not want to see the likelihood function).
- Its main benefit is its invariance to the normalization of the equation (Greene 2008: 375).

Let's try using 3SLS

```
. est table P_OLS P_2SLS  Three, b(%9.5f) se

--------------------------------------------------
    Variable |   P_OLS        P_2SLS       Three
-------------+------------------------------------
_            |
     gdpenl  |   0.63203      1.41713
             |   0.02058      0.08169
     colbrit |   2.19211      0.92052
             |   0.19898      0.25479
      mtnest |   0.00655      0.01515
             |   0.00414      0.00467
         Oil |  -5.10208     -7.32394
             |   0.26408      0.36768
          ef |  -4.43352     -1.12538
             |   0.34296      0.50411
        warl |   1.55630      2.80552
             |   0.25690      0.31127
       _cons |  -0.83291     -4.92194
             |   0.21460      0.47286
-------------+------------------------------------
polity2      |
      gdpenl |                            1.46536
             |                            0.08137
     colbrit |                            2.18466
             |                            0.23449
      mtnest |                            0.00839
             |                            0.00420
         Oil |                           -7.17073
             |                            0.36703
          ef |                           -3.82107
             |                            0.48275
        warl |                            1.52645
             |                            0.28745
       _cons |                           -3.94269
             |                            0.46807
-------------+------------------------------------
gdpenl       |
         Oil |                            3.43087
             |                            0.16846
        year |                            0.05038
             |                            0.00353
      muslim |                           -0.03048
             |                            0.00144
     relfrac |                            0.44866
             |                            0.23119
       _cons |                          -95.71524
             |                            6.96376
--------------------------------------------------
```

Notice the slightly larger coefficient.

- Let’s get back to our dichotomous variable problem.
- Suppose that we have one dichotomous variable and one continuous variable.

- Going back to our old two endogenous variable model:

$$Y_1 = \beta_{01} + \beta_{11}Y_2 + \boldsymbol{\beta_m X_m} + \varepsilon_1$$
$$Y_2 = \beta_{02} + \beta_{12}Y_1 + \boldsymbol{\beta_k X_k} + \varepsilon_2$$

- Let's assume that Y_1 is observed dichotomously, where we are interested in the latent continuous variable Y_1*:

$$Y_{1i} = \begin{cases} 0 \; if \; Y^{1*} < 0 \\ 1 \; if \; Y^{1*} \geq 0 \end{cases}$$

- This multi-equation instrumental variable model can now be estimated using maximum likelihood.

The ivprobit likelihood function

$$\ln L_i = w_i \left[y_{1i} \ln \Phi\left(m_i\right) + \left(1 - y_{1i}\right) \ln \left\{1 - \Phi\left(m_i\right)\right\} + \ln \phi \left(\frac{y_{2i} - \boldsymbol{x}_i \boldsymbol{\Pi}}{\sigma} \right) - \ln \sigma \right]$$

where

$$m_i = \frac{\boldsymbol{z}_i \boldsymbol{\delta} + \rho \left(y_{2i} - \boldsymbol{x}_i \boldsymbol{\Pi}\right) / \sigma}{\left(1 - \rho^2\right)^{\frac{1}{2}}}$$

- From *Stata 11 Base Reference Manual*: 733.
- Let's try this function on some real data.

Keshk, Pollins, and Reuveny (2004)

- Keshk, Omar M.G., Brian M. Pollins, & Rafael Reuveny. (2004) "Trade Still Follows the Flag: The Primacy of Politics in a Simultaneous Model of Interdependence and Armed Conflict," *Journal of Politics*, 66(4).
- This article models the interrelationship of trade and conflict.
- Their measure of trade is continuous, but their measure of conflict is dichotomous.
- What to do, what to do?

```
. ivprobit dispute dispute_lag dependence lower_growth lower_democracy alliances capability_ratio //
>  (trade = trade_lag gdp_A gdp_B pop_A pop_B distance lower_democracy alliances ), robust cluster(cluster)

Fitting exogenous probit model

Iteration 6:   log likelihood = -3500.0329

Fitting full model
Iteration 3:   log pseudolikelihood = -266825.03

Probit model with endogenous regressors           Number of obs   =     143792
                                                  Wald chi2(7)    =    1676.10
Log pseudolikelihood = -266825.03                 Prob > chi2     =     0.0000

                             (Std. Err. adjusted for 6636 clusters in cluster)
------------------------------------------------------------------------------
             |               Robust
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       trade |   .0382846   .0038199    10.02   0.000     .0307978    .0457714
 dispute_lag |   2.480361   .0833551    29.76   0.000     2.316988    2.643734
  dependence |  -59.12915   27.28599    -2.17   0.030    -112.6087   -5.649594
lower_growth |  -.0073218   .0041565    -1.76   0.078    -.0154684    .0008248
lower_demo~y |  -.1500483   .0205881    -7.29   0.000    -.1904003   -.1096964
   alliances |   .3252931   .0551563     5.90   0.000     .2171888    .4333973
capability~o |  -.0001679   .0001581    -1.06   0.289    -.0004778    .0001421
       _cons |  -2.852112   .0481366   -59.25   0.000    -2.946458   -2.757766
-------------+----------------------------------------------------------------
     /athrho |  -.1284848   .0190344    -6.75   0.000    -.1657914   -.0911781
    /lnsigma |    .412911   .0063298    65.23   0.000     .4005047    .4253172
-------------+----------------------------------------------------------------
         rho |  -.1277824   .0187236                     -.1642889   -.0909263
       sigma |    1.51121   .0095657                      1.492578    1.530076
------------------------------------------------------------------------------
Instrumented:  trade
Instruments:   dispute_lag dependence lower_growth lower_democracy alliances
               capability_ratio trade_lag gdp_A gdp_B pop_A pop_B distance
------------------------------------------------------------------------------
Wald test of exogeneity (/athrho = 0): chi2(1) =    45.56 Prob > chi2 = 0.0000
```

- Stata conducts a Wald Chi-squared test of the null hypothesis that there is no significant endogeneity between trade and conflict.

- Clearly, our results suggest significant endogeneity.

- What if one of our continuous variables we used in the democracy and development models was truncated?

- We could use instrumental variable tobit developed by Newey (1987).

- Similar to ivprobit, we are trying to estimate the effect of a continuous variable that is only partially observed:

$$Y_{1i} = \begin{cases} a & if\ Y^{1*} < a \\ Y^{1*} & if\ \ a \leq Y^{1*} \leq b \\ b & if\ Y^{1*} > b \end{cases}$$

Ivtobit likelihood function (*Stata reference*: 783)

$$\ln f\left(y_{2i}|\boldsymbol{x}_i\right) = -\frac{1}{2}\left\{\ln 2\pi + \ln\sigma_v^2 + \frac{\left(y_{2i} - \boldsymbol{x}_i\boldsymbol{\Pi}\right)^2}{\sigma_v^2}\right\}$$

and

$$\ln f\left(y_{1i}|y_{2i}, \boldsymbol{x}_i\right) = \begin{cases} \ln\left\{1 - \Phi\left(\frac{m_i - a}{\sigma_{u|v}}\right)\right\} & y_{1i} = a \\ -\frac{1}{2}\left\{\ln 2\pi + \ln\sigma_{u|v}^2 + \frac{\left(y_{1i} - m_i\right)^2}{\sigma_{u|v}^2}\right\} & a < y_{1i} < b \\ \ln\Phi\left(\frac{m_i - b}{\sigma_{u|v}}\right) & y_{1i} = b \end{cases}$$

where

$$m_i = \boldsymbol{z}_i\boldsymbol{\delta} + \alpha\left(y_{2i} - \boldsymbol{x}_i\boldsymbol{\Pi}\right)$$

- Going back to the Fearon and Laitin (2003) data on development and democracy.

- Suppose we truncate logged trade at -4 (trade ranges from -5 to 26.9).

- Going back to the Fearon and Laitin (2003) data on development and democracy.

- Suppose we truncate logged trade at -4.
  - Trade ranges from -5 to 26.9.

```
. ivtobit polity2 colbrit mtnest Oil ef warl (gdpenl = Oil year muslim relfrac), ///
>  first robust cluster(ccode) ll(-4) nolog
Tobit model with endogenous regressors              Number of obs   =       6243
                                                    Wald chi2(6)    =      19.29
Log pseudolikelihood = -30964.124                   Prob > chi2     =     0.0037
                                (Std. Err. adjusted for 153 clusters in ccode)
------------------------------------------------------------------------------
             |               Robust
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
polity2      |
      gdpenl |   4.190278   2.644885     1.58   0.113    -.9936012    9.374157
     colbrit |  -2.020339   5.978189    -0.34   0.735    -13.73737    9.696695
      mtnest |   .0562782   .0542514     1.04   0.300    -.0500526     .162609
         Oil |  -16.65878   10.67326    -1.56   0.119    -37.57798     4.26043
          ef |   7.434987   10.60249     0.70   0.483    -13.34551    28.21549
        warl |   7.332556   5.117959     1.43   0.152     -2.69846    17.36357
       _cons |  -20.21693   13.42799    -1.51   0.132    -46.53531    6.101455
-------------+----------------------------------------------------------------
gdpenl       |
     colbrit |   1.771894   .7948861     2.23   0.026     .2139456    3.329842
      mtnest |  -.0105558   .0149111    -0.71   0.479     -.039781    .0186694
         Oil |   3.556262   .9057549     3.93   0.000     1.781015    5.331509
          ef |  -3.660252   .9242239    -3.96   0.000    -5.471698   -1.848806
        warl |  -1.618833   .5502685    -2.94   0.003     -2.69734   -.5403268
        year |   .0255598   .0241636     1.06   0.290    -.0218001    .0729197
      muslim |  -.0231628   .0078769    -2.94   0.003    -.0386013   -.0077243
     relfrac |  -.1607206   1.157864    -0.14   0.890    -2.430093    2.108652
       _cons |  -45.05744   47.74045    -0.94   0.345     -138.627    48.51213
-------------+----------------------------------------------------------------
      /alpha |  -3.513232   2.748541    -1.28   0.201    -8.900273    1.873809
        /lns |   2.115037   .0550869    38.39   0.000     2.007069    2.223006
        /lnv |   1.392301   .1398259     9.96   0.000     1.118247    1.666355
-------------+----------------------------------------------------------------
           s |   8.289896   .4566647                      7.441475    9.235048
           v |   4.024099   .5626734                      3.059487     5.29284
------------------------------------------------------------------------------
Instrumented:  gdpenl
Instruments:   colbrit mtnest Oil ef warl year muslim relfrac
------------------------------------------------------------------------------
Wald test of exogeneity (/alpha = 0): chi2(1) =     1.63  Prob > chi2 = 0.2012

  Obs. summary:       3011  left-censored observations at polity2<=-4
                      3232     uncensored observations
                         0 right-censored observations
```

In summary

- These models (2SLS, 3SLS, ivprobit, and ivtobit) represent some of the most common multiple equation models besides the selection models we have seen in earlier weeks.
- However, there are numerous other models that scholars have developed to empirically model what they have argued theoretically.
- For today I have had you read two such efforts—Clark and Reed (2005) and Reuveny and Lai (2003).
- One of which has nothing but dichotomous dependent variables.

Reuveny and Lai (2003)

- Three equations using 2SLS.

$$DEM_L = \beta_0 + \beta_L M_L + \beta_{ABU} MID_{ABU} + \beta_{LRU} MID_{LRU} + \varepsilon_L$$

$$DEM_H = \theta_0 + \theta_H M_H + \theta_{ABU} MID_{ABU} + \theta_{LRU} MID_{HRU} + \varepsilon_H$$

$$MID_{ABU} = \zeta_0 + \zeta_{AB} X_{AB} + \zeta_L DEM_L + \zeta_H DEM_H + \varepsilon_{ABU}$$

Clark and Reed (2005)

- A more complex multiple equation model also with three equations with dichotomous dependent variables.

- *Equation 1*: Is the US targeted?
- *Equation 2*: US respond with sanctions?
- *Equation 3*: US respond with force?

Clark and Reed (2005)

- Also written as:

$$Y_1 = \beta_0 + \beta_{US}X_{US} + \beta_{US}X_B + \varepsilon_1$$

$$Y_2 = \theta_0 + \theta_{US}X_{US} + \theta_B X_B + \theta_T(Y_1) + \varepsilon_2$$

$$Y_3 = \zeta_0 + \zeta_{US}X_{US} + \zeta_B X_B + \zeta_T(Y_1) + \varepsilon_3$$

- Clark and Reed (2005) are therefore modeling a selection equation with two selection mechanisms
  - 1. A state decides to target the US.
  - 2. The US decides how to respond (sanctions or force)
- How are the error structures of the three equations related?
- Is this a recursive or non-recursive model?

- How do they estimate the model?

- GHK smooth recursive simulated bivariate probit models (Cappellari and Jenkins 2003; Greene 2008: 823-831).

- Whew, that sounds complicated!

- Cappellari and Jenkins (2003) have made this easily runnable in an ado file (`mvprobit`).

Let's take a step back...

- Multiple equation models are useful tools for modeling more complex theoretical models where we have reason to believe that effects are not only in one direction.
- They can be estimated using almost every model we have seen previously in this class.
- They require thinking about the **distributions of our errors**.
- Often it can be **difficult to find appropriate instruments** for identifying your models.
- That is why having a **strong theoretical foundation** is so important.