Week 15

Censored and Truncated Variables

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Today's Stata tips

- The Stata listserv is a useful way to learn new techniques in Stata (and ask questions).

- Also, Stata has a page of replication data for all Stata publications <http://www.stata-press.com/data/books.html>.

Today

- Modeling censoring and truncation!
- Both deal with sample data that are not randomly drawn from the population.
- How does the data-generating process lead to empirical consequences that we need to take into account?

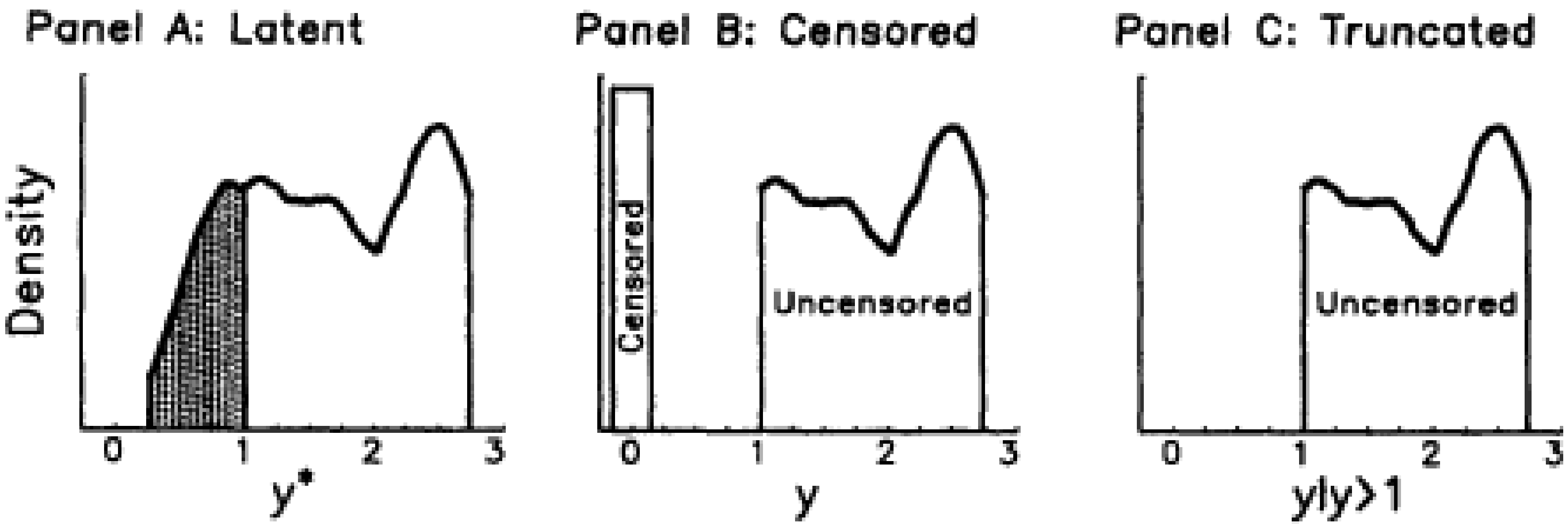
- We have already seen censoring and truncation in the context of discrete dependent variables like event counts (e.g. ZIP & ZINB).

- Today we focus on truncation and censoring of otherwise *continuous* dependent variables

I. Truncation

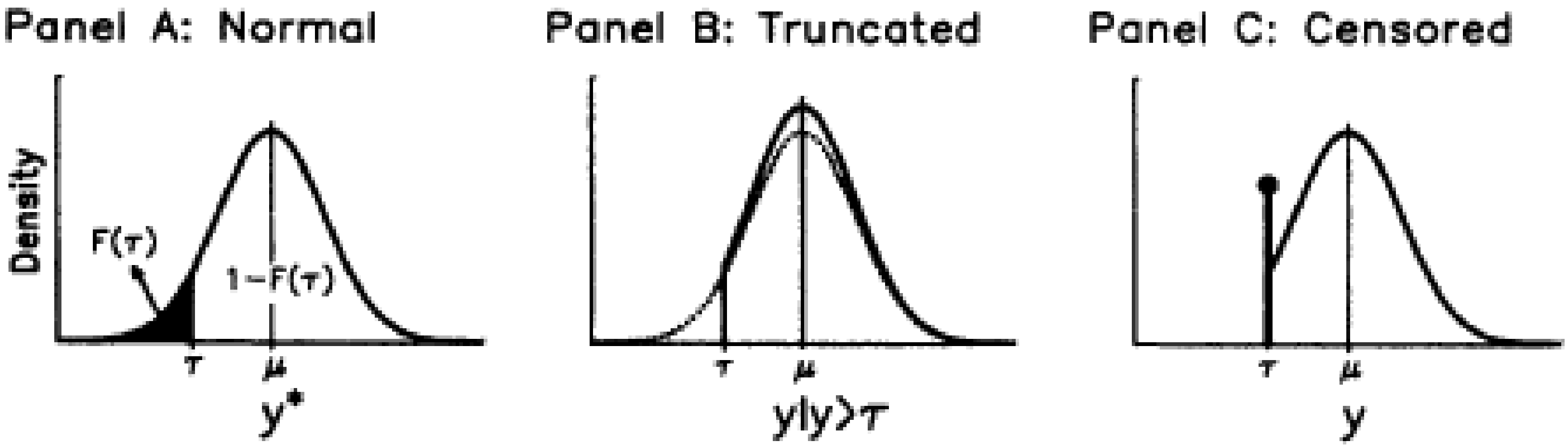
- A truncated variable is one that represents only part of a distribution.
- This truncation leads to observations where the observed variable y has values above a certain threshold are excluded from the sample.

Long 1997

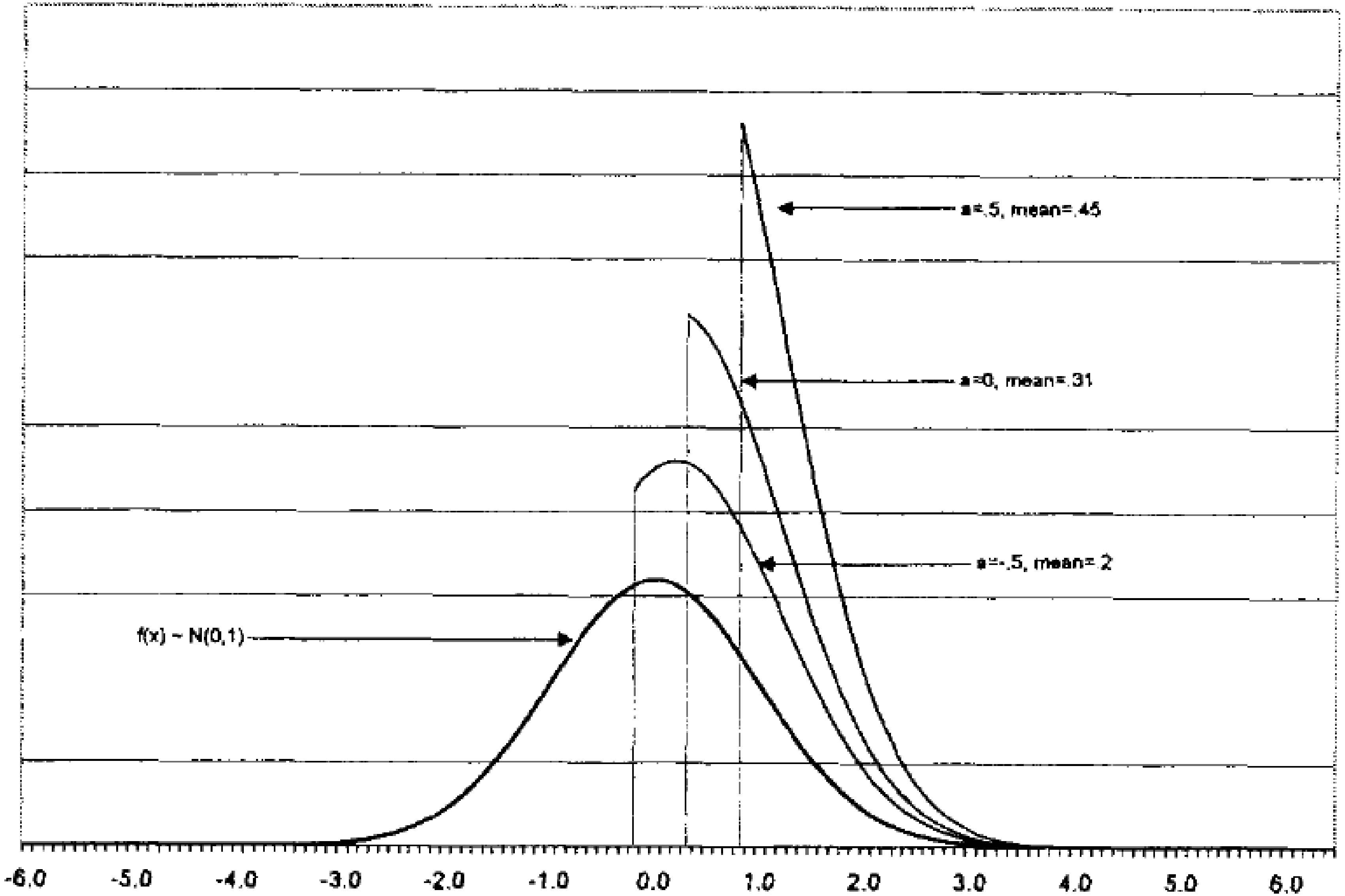


Panel A: Latent
Density
0
1
2
3
y*
Panel B: Censored
Censored
Uncensored
0
1
2
3
y
Panel C: Truncated
Uncensored
0
1
2
3
y|y>1

Long 1997



Panel A: Normal
Density
F(τ)
1−F(τ)
τ
μ
y*
Panel B: Truncated
τ
μ
y|y>τ
Panel C: Censored
τ
μ
y


a=.5, mean=.45
a=0, mean=.31
a=-.5, mean=.2
f(x) ~ N(0,1)
-6.0
-5.0
-4.0
-3.0
-2.0
-1.0
0.0
1.0
2.0
3.0
4.0
5.0
6.0

- The density of a truncated continuous variable is:

$$f(x|\, x > a) = \frac{f(x)}{P(x > a)}$$

- The distribution of *x* is conditional on the probability that its value is above *a*.
  - *a* is also referred in other works as τ and κ, but what is important to remember is that it is referring to the cut point where truncation or censoring occurs.

- This would be left-truncation.

- If *x* is observed if *x* < *a* then it would be right-truncation at *a*.

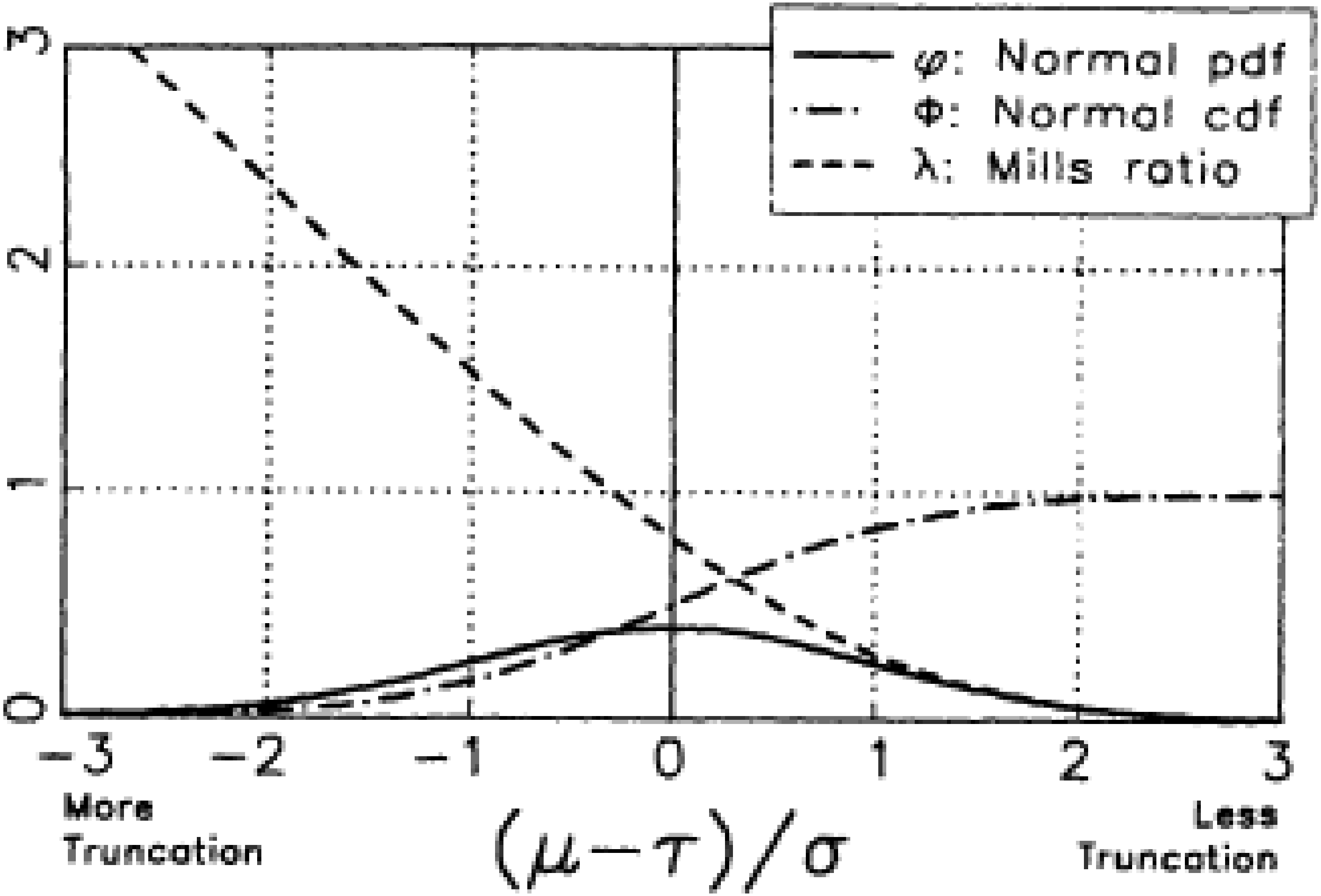
Inverse Mills Ratio

- This is the ratio between the PDF and CDF.

$$\lambda(y^*) = \frac{\phi(y^*)}{\Phi(y^*)}$$

- Why is the Inverse Mills Ratio important?
  - It represents the number of standard deviations that the mean is above or below the truncation point.

Long 1997


φ: Normal pdf
Φ: Normal cdf
λ: Mills ratio
0
1
2
3
−3
−2
−1
0
1
2
3
More Truncation
$(\mu - \tau)/\sigma$
Less Truncation

Truncated normal distribution

- This is truncation from below:

$$E(y\,|a) = \mu + \sigma \left[\frac{\phi\left(\frac{a-\mu}{\sigma}\right)}{1-\Phi\left(\frac{a-\mu}{\sigma}\right)} \right]$$

- Greene (2007) simplifies this to:

$$\alpha = \frac{a-\mu}{\sigma}$$

$$\lambda(\propto) = \frac{\phi(\alpha)}{1-\Phi(\alpha)}$$

- So:

$$E(y\,|a) = \mu + \sigma\lambda(a)$$

Which can be parameterized

$$E(y\,|\,y > a) = \beta x + \sigma\left(\frac{\phi(\beta x)/\sigma}{\Phi(\beta x)/\sigma}\right)$$

- This means that the mean of the distribution will be *larger* than if there was no truncation.
  - The less the truncation the closer the E(y) will be to μ.

- Alternatively, right-truncation lowers the mean.

- Also, the variance would be smaller than the full distribution.

- This is because:

$$Var(y|a) = \sigma^2[1 - \delta(a)]$$

$$\delta(a) = \lambda(a)[\lambda(a) - a]$$

$$0 < \delta(a) < 1$$

- Therefore the truncated variance is equal to the untruncated variance weighted by $[1 - \delta(a)]$.

Truncated models in Stata

- Command: `truncreg`
- Can model both right and left-truncation.

```
. truncreg jobcen1 $xlist if jobcen1>1, ll(1)
(note: 0 obs. truncated)

Fitting full model:

Iteration 0:   log likelihood = -319.95029
Iteration 1:   log likelihood = -318.66838
Iteration 2:   log likelihood = -318.66025
Iteration 3:   log likelihood = -318.66024

Truncated regression
Limit:   lower =          1                     Number of obs =    309
         upper =       +inf                     Wald chi2(6)  =  71.13
Log likelihood = -318.66024                     Prob > chi2   = 0.0000

------------------------------------------------------------------------------
     jobcen1 |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         fem |    .114156    .095124     1.20   0.230    -.0722837    .3005956
         phd |   .3413744   .0539561     6.33   0.000     .2356224    .4471263
        ment |   .0008171   .0006589     1.24   0.215    -.0004743    .0021085
         fel |   .1709118   .1011169     1.69   0.091    -.0272737    .3690974
         art |   .0072712   .0271957     0.27   0.789    -.0460314    .0605738
         cit |   .0021862    .001788     1.22   0.221    -.0013182    .0056905
       _cons |   1.187784   .1962769     6.05   0.000     .8030885     1.57248
-------------+----------------------------------------------------------------
      /sigma |   .7379857   .0353198    20.89   0.000     .6687602    .8072112
------------------------------------------------------------------------------
```

Comparing truncreg and regress results

```
. estout truncreg ols1 , cells(b(star fmt(3)) se(par)) stats(r2 N aic)
--------------------------------------------
                 truncreg         ols1
                     b/se         b/se
--------------------------------------------
main
fem                 0.114       -0.139
                  (0.095)      (0.090)
phd                 0.341***     0.273***
                  (0.054)      (0.049)
ment                0.001        0.001
                  (0.001)      (0.001)
fel                 0.171        0.234*
                  (0.101)      (0.095)
art                 0.007        0.023
                  (0.027)      (0.029)
cit                 0.002        0.004*
                  (0.002)      (0.002)
_cons               1.188***     1.067***
                  (0.196)      (0.166)
--------------------------------------------
sigma
_cons               0.738***
                  (0.035)
--------------------------------------------
r2                               0.210
N                 309.000      408.000
aic               653.320     1052.793
```

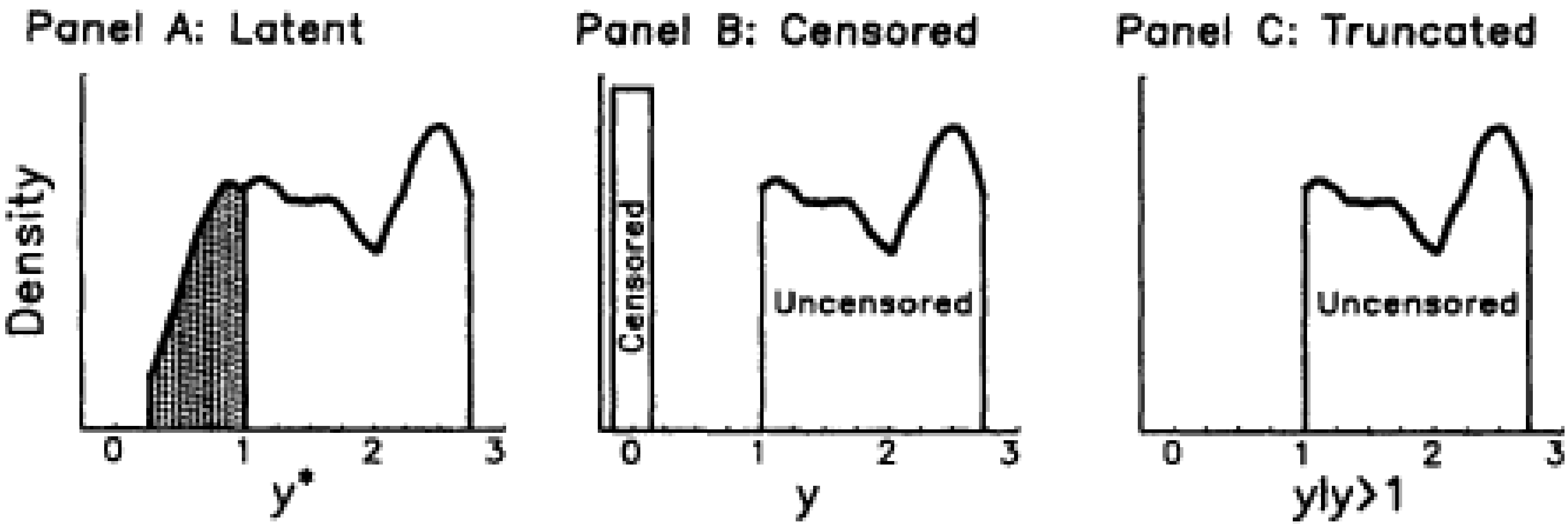
II. Censored variables

- What is the difference between censored and truncated data?

Censored variables

- Censored variables do not drop observations if they do not experience the phenomenon of interest.
  - For example, how much I spend per month on cigarettes.
- Rather they are coded as taking a limited or threshold value ($0).

Remember…



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y
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0
1
2
3
y|y>1

Other censored variables examples

- Household purchases of durable goods (Tobin 1958).
- Vacation expenditures (Melenburg and van Soest 1996)
- How do we find those PDF and CDF distributions for censored variables?

Censored Normal Distribution

$$P(y = 0) = P(y^* \leq 0) = \Phi\left(\frac{\mu}{\sigma}\right)$$

- The probability of non-limited observations is a density for $y^* > 0$, so y has a density of y^*.

$$E(y|\, a = 0) = \Phi\left(\frac{\mu}{\sigma}\right)\left(\mu + \sigma\lambda\left(\frac{\mu}{\sigma}\right)\right)$$

- Again, remember that λ is analogous to the Mills ratio.

$$\lambda = \frac{\phi(.)}{\Phi(.)}$$

$$\lambda = \frac{\phi(\frac{\mu}{\sigma})}{\Phi(\frac{\mu}{\sigma})}$$

For right-censored data

$$P(Censored) = P(y = 0) = P(y^* \leq 0)$$

$$= \Phi\left(\frac{a - \mu}{\sigma}\right)$$

$$P(Uncensored) = P(y = y^*) = P(y^* > 0)$$

$$= \Phi\left(\frac{\mu - a}{\sigma}\right)$$

- Combining both of these equations:

$$E(y|a) = \Phi\left(\frac{\mu - a}{\sigma}\right)\left[\mu + \sigma\lambda\left(\frac{\mu - a}{\sigma}\right)\right] + \Phi\left(\frac{a - \mu}{\sigma}\right)\mathrm{a}$$

- As you can see if the censoring value (a) equals 0 then it reduces to the uncensored probability.

- Can you visualize the standard normal curve here?

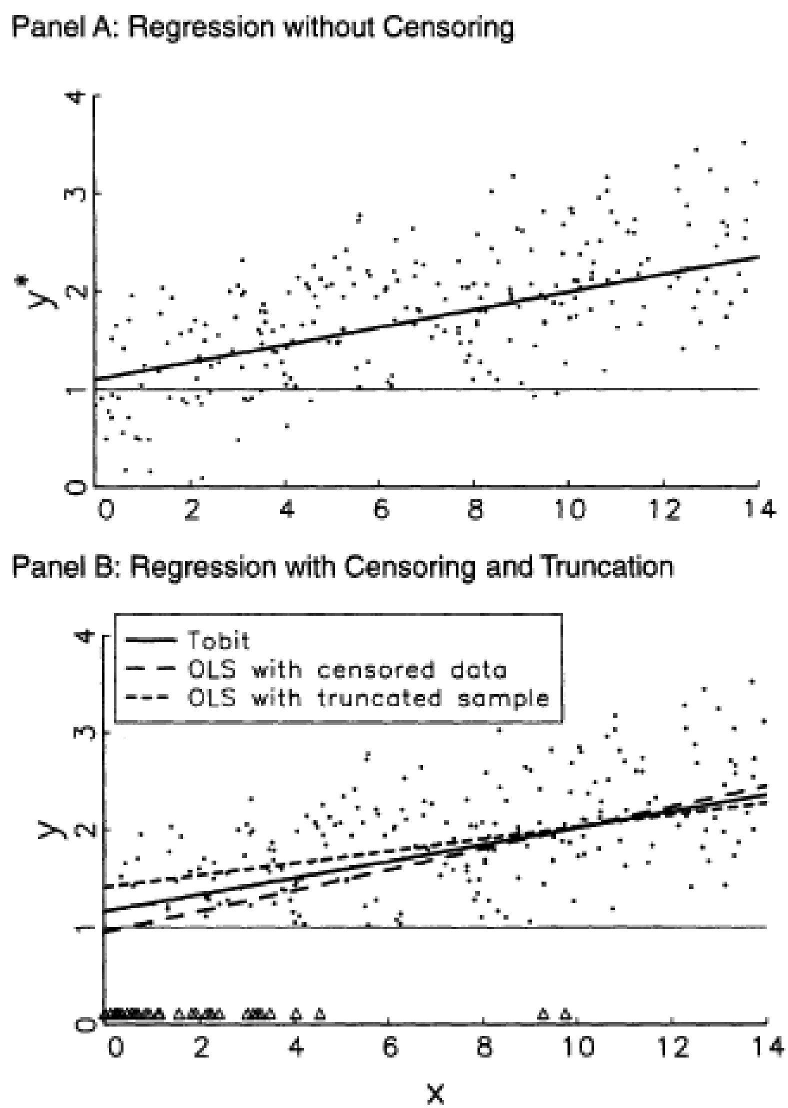
- The log-likelihood is given by:

$$\ln(L) =$$

$$\sum_{uncensored}^{y_i|\, y_i>0} ln \frac{1}{\sigma} \phi \left(\frac{y_i - x_i\beta}{\sigma} \right) +$$

$$\sum_{censored}^{y_i|\, y_i=0} ln\Phi \left(\frac{a \;-\; x_i\beta}{\sigma} \right)$$

Accounting for censoring can affect your substantive conclusions



Sigelman and Zeng (1999: 176)

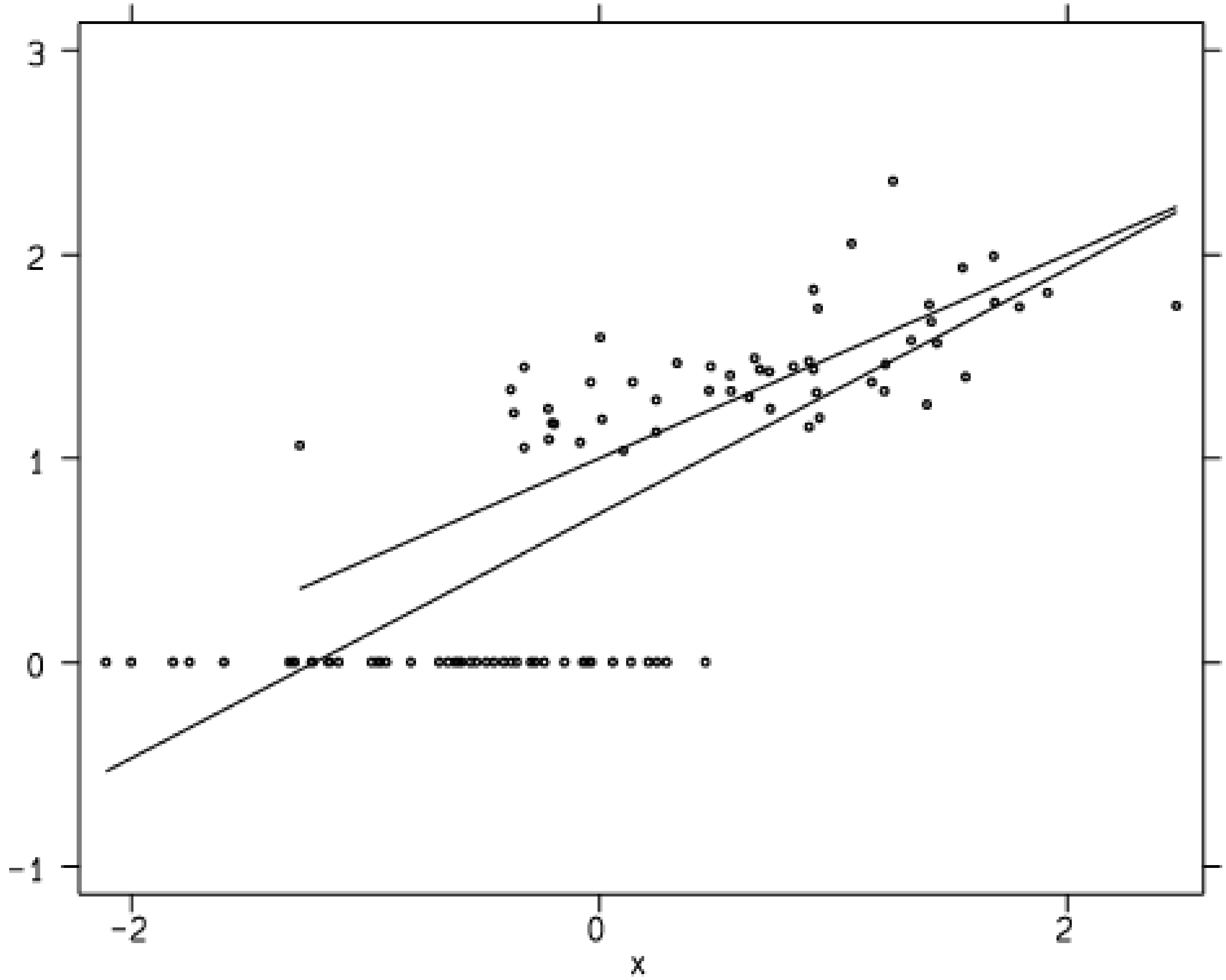


Fig. 2 Upward bias of OLS estimates on censored data.

Sigelman and Zeng (1999: 176)

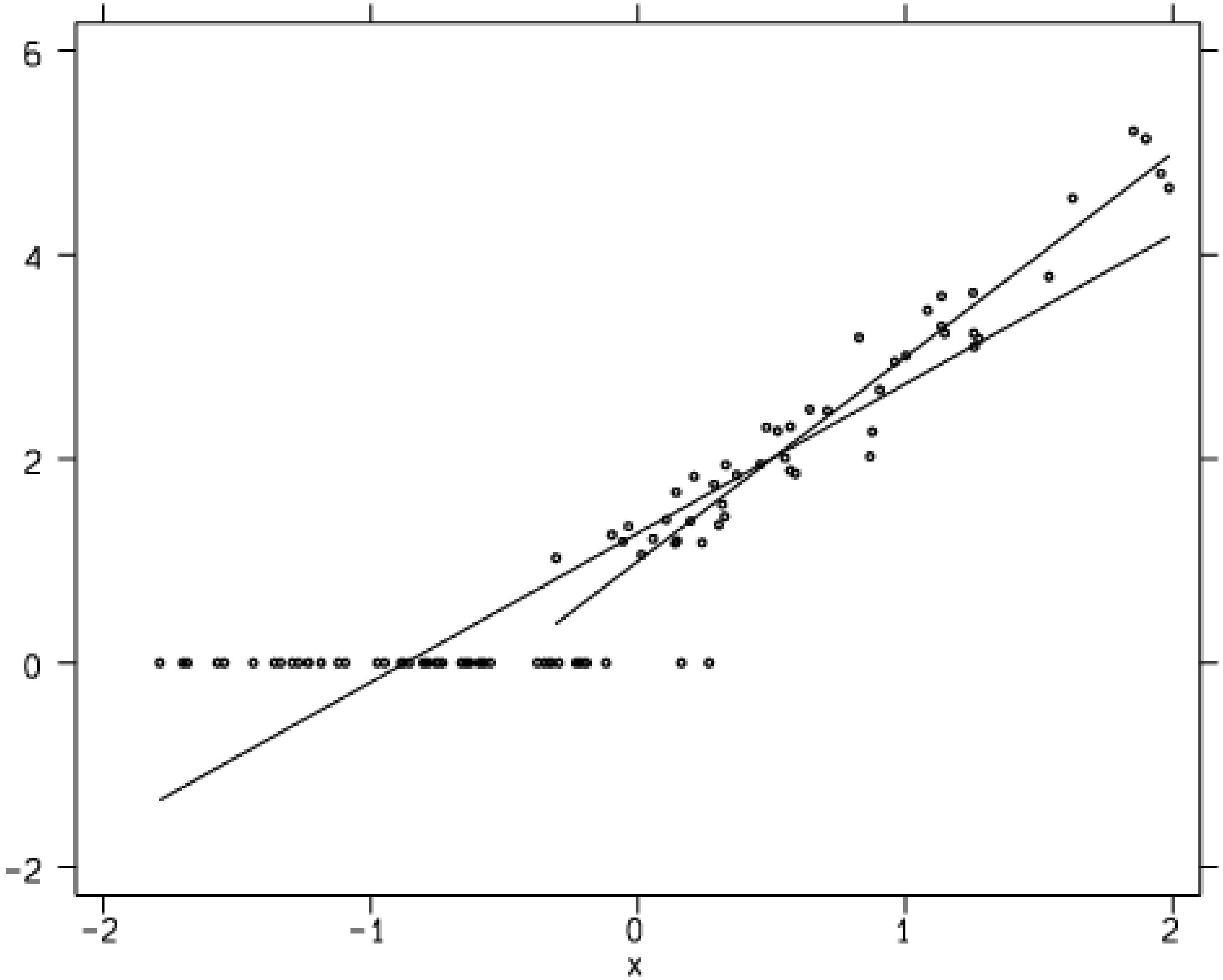


Fig. 1 Downward bias of OLS estimates on censored data.

Tobit censored regression

- Named in reference to Tobin (1958) who first proposed it.
- Tobin's technique allows us to derive consistent and asymptotically efficient estimators given right or left censoring.

Estimates the joint probability of censored and uncensored observations

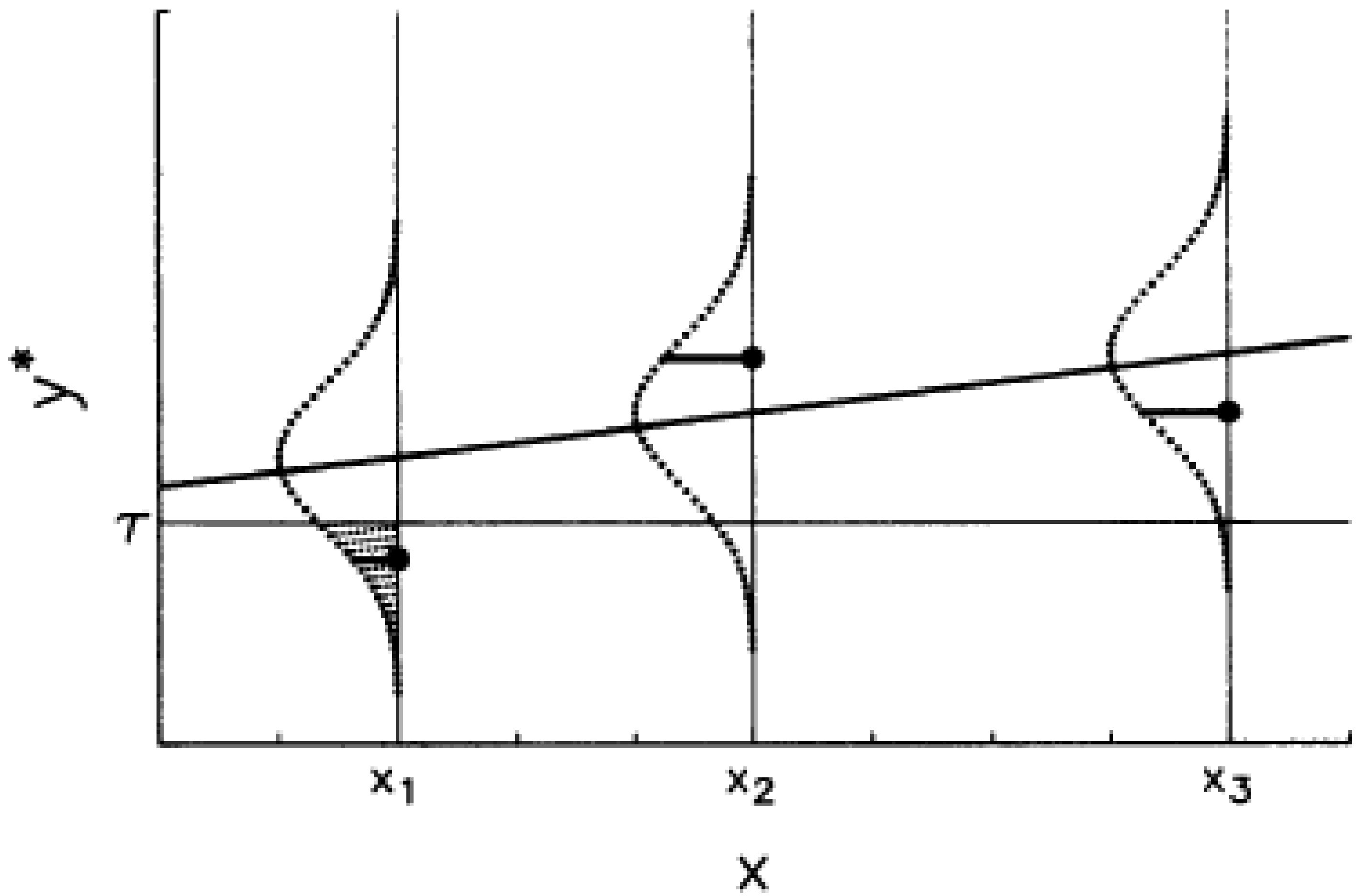


Figure 7.8. Maximum Likelihood Estimation for the Tobit Model

Tobit Likelihood function

- This function estimates the joint probability of the censored and uncensored observations (Greene 2008: 874).

- Similar to the censored regression

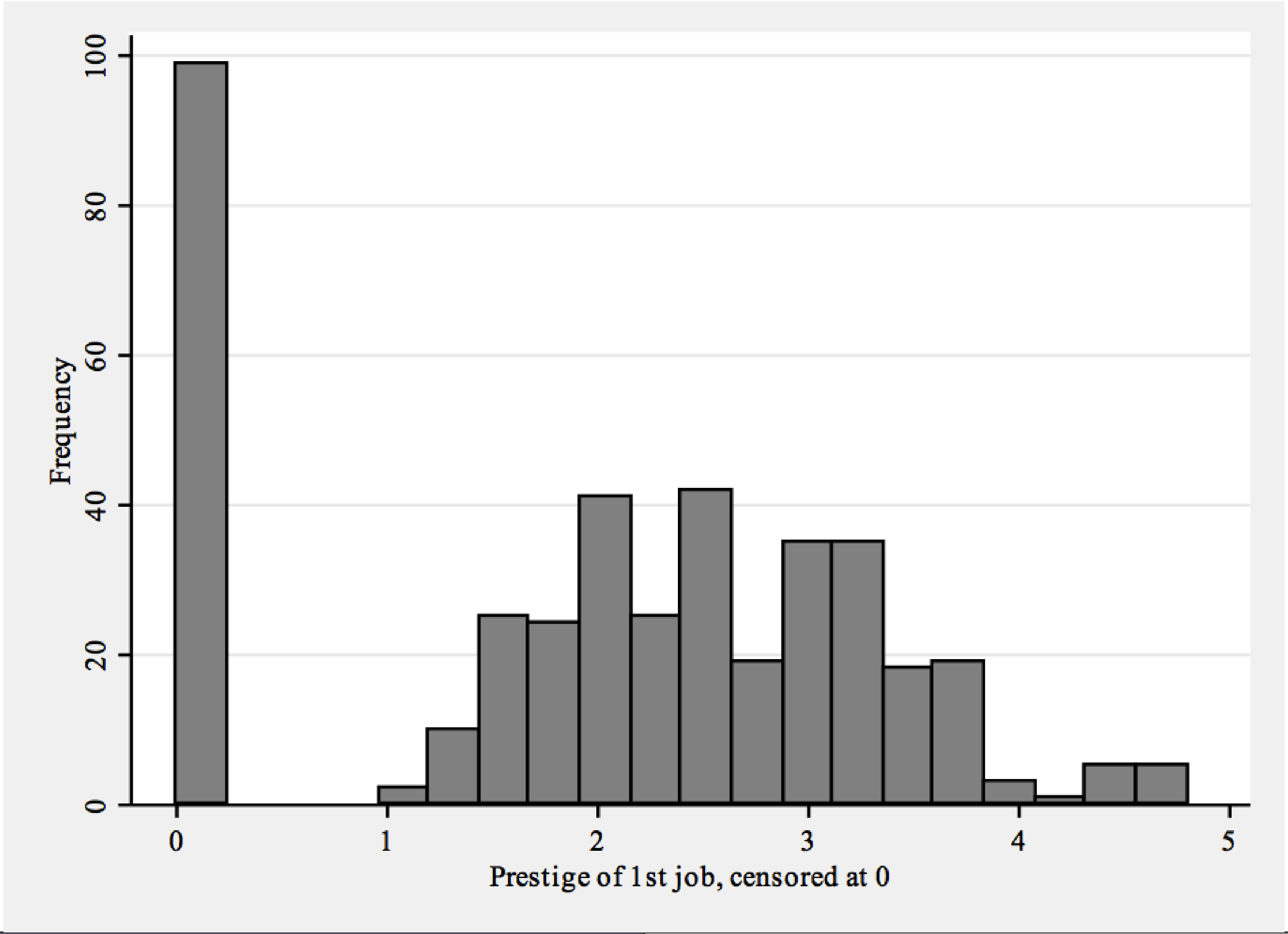
$$\ln(L) = \sum_{y_i>0} -\frac{1}{2}\left[log(2\pi) + ln\sigma^2 + \frac{(y_i - \beta x_i)^2}{\sigma^2}\right] + \sum_{y_i=0} ln\left[1 - \Phi\left(\frac{\beta x_i}{\sigma}\right)\right]$$

Example: First job prestige

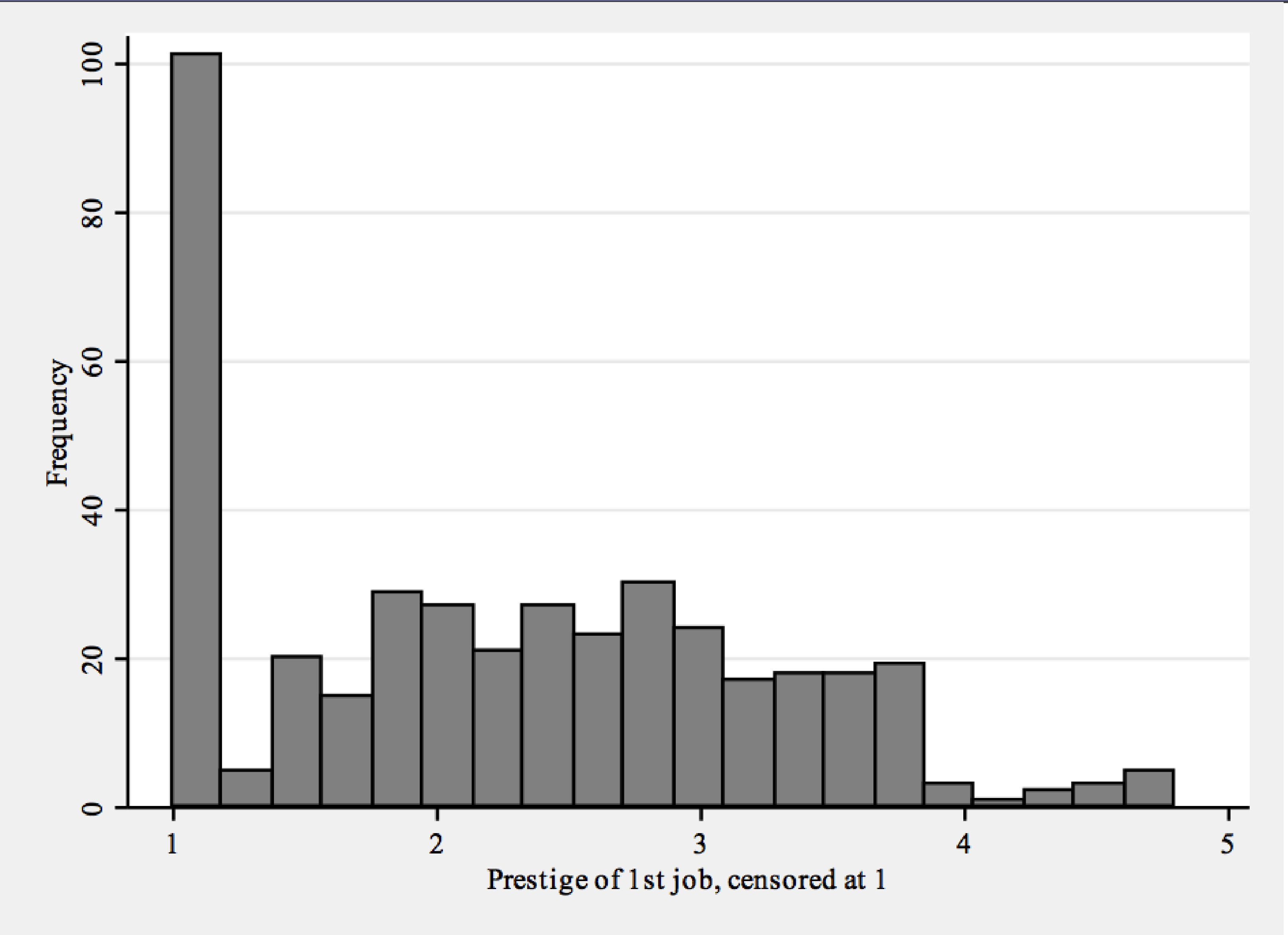
- Data from Long (1997)

- Dependent variable prestige of first job is censored below 1 because departments under 1 were not rated along with departments that did not have graduate programs.

Censored at 0



Censored at 1



OLS Results

```
. global xlist fem phd ment fel art cit

. set more off

. reg jobcen1 $xlist

      Source |       SS       df       MS              Number of obs =     408
-------------+------------------------------           F(  6,   401) =   17.78
       Model |  81.0584763     6  13.5097461           Prob > F      =  0.0000
    Residual |  304.737915   401  .759944926           R-squared     =  0.2101
-------------+------------------------------           Adj R-squared =  0.1983
       Total |  385.796392   407  .947902683           Root MSE      =  .87175

------------------------------------------------------------------------------
     jobcen1 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         fem |  -.1391939   .0902344    -1.54   0.124    -.3165856    .0381977
         phd |   .2726826   .0493183     5.53   0.000     .1757278    .3696375
        ment |   .0011867   .0007012     1.69   0.091    -.0001917    .0025651
         fel |   .2341384   .0948206     2.47   0.014     .0477308    .4205461
         art |   .0228011   .0288843     0.79   0.430    -.0339824    .0795846
         cit |   .0044788   .0019687     2.28   0.023     .0006087     .008349
       _cons |   1.067184   .1661357     6.42   0.000     .7405785     1.39379
------------------------------------------------------------------------------
```

Tobit, censored at 0

```
. tobit jobcen0 $xlist, ll(0)

Tobit regression                                  Number of obs   =        408
                                                  LR chi2(6)      =      78.65
                                                  Prob > chi2     =     0.0000
Log likelihood = -668.85727                       Pseudo R2       =     0.0555

------------------------------------------------------------------------------
     jobcen0 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         fem |  -.4026382   .1609166    -2.50   0.013    -.7189814    -.086295
         phd |   .3714449   .0881261     4.21   0.000     .1981993    .5446906
        ment |   .0016541   .0012284     1.35   0.179    -.0007609     .004069
         fel |   .4478896   .1689947     2.65   0.008     .1156659    .7801133
         art |   .0509334    .050508     1.01   0.314    -.0483595    .1502262
         cit |   .0065408   .0034275     1.91   0.057    -.0001973    .0132788
       _cons |   .1480861   .3001994     0.49   0.622    -.4420708     .738243
-------------+----------------------------------------------------------------
      /sigma |   1.506908   .0652557                      1.378623    1.635193
------------------------------------------------------------------------------
  Obs. summary:         99  left-censored observations at jobcen0<=0
                       309     uncensored observations
                         0 right-censored observations
```

Tobit, censored at 1

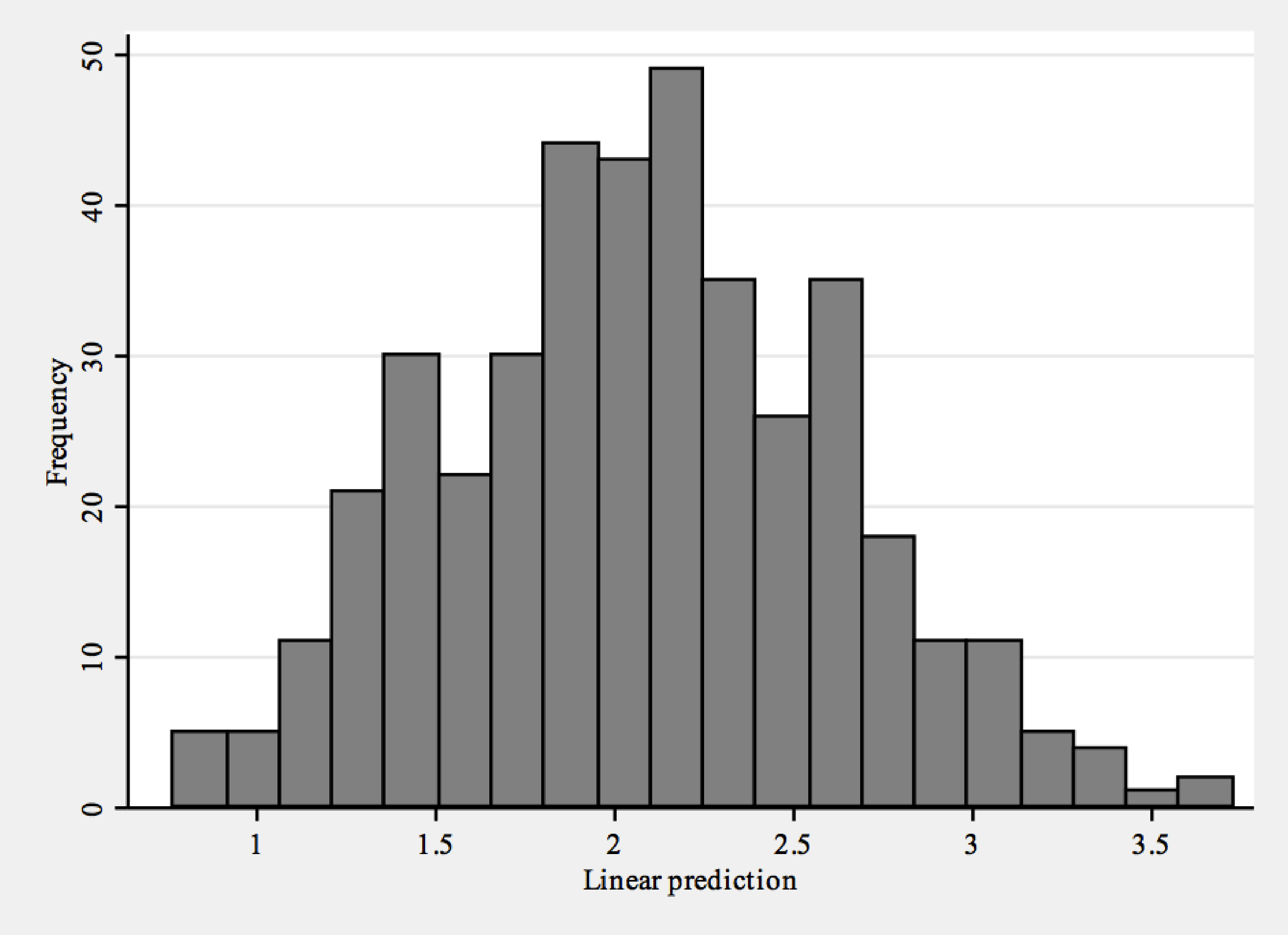
```
. tobit jobcen1 $xlist, ll(1)

Tobit regression                                  Number of obs   =        408
                                                  LR chi2(6)      =      89.20
                                                  Prob > chi2     =     0.0000
Log likelihood = -560.25209                       Pseudo R2       =     0.0737

------------------------------------------------------------------------------
     jobcen1 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         fem |  -.2368486   .1165795    -2.03   0.043    -.4660302   -.0076669
         phd |   .3225846   .0639198     5.05   0.000     .1969258    .4482435
        ment |   .0013436   .0008875     1.51   0.131    -.0004011    .0030884
         fel |   .3252657   .1224516     2.66   0.008     .0845403    .5659912
         art |   .0339053      .0365     0.93   0.353    -.0378493      .10566
         cit |     .00509   .0024751     2.06   0.040     .0002243    .0099557
       _cons |   .6854061    .218261     3.14   0.002     .2563306    1.114482
-------------+----------------------------------------------------------------
      /sigma |   1.087237    .046533                      .9957585    1.178715
------------------------------------------------------------------------------
  Obs. summary:         99  left-censored observations at jobcen1<=1
                       309     uncensored observations
                         0 right-censored observations
```

```
. estout ols0 ols0c ols1 ols1c tobit0 tobit1, cells(b(star fmt(3)) se(par)) ///
>    stats(r2 N aic)
```

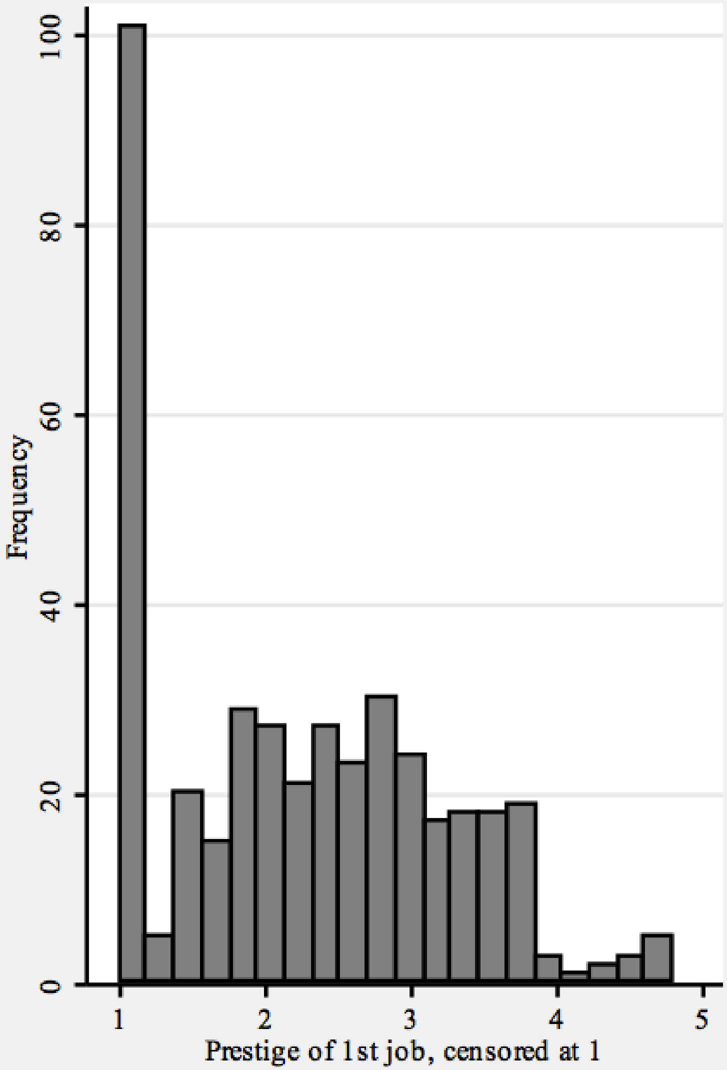
	ols0	ols0c	ols1	ols1c	tobit0	tobit1
	b/se	b/se	b/se	b/se	b/se	b/se
main						
fem	-0.274*	0.101	-0.139	0.101	-0.403*	-0.237*
	(0.123)	(0.085)	(0.090)	(0.085)	(0.161)	(0.117)
phd	0.314***	0.297***	0.273***	0.297***	0.371***	0.323***
	(0.067)	(0.047)	(0.049)	(0.047)	(0.088)	(0.064)
ment	0.001	0.001	0.001	0.001	0.002	0.001
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
fel	0.335*	0.141	0.234*	0.141	0.448**	0.325**
	(0.130)	(0.090)	(0.095)	(0.090)	(0.169)	(0.122)
art	0.037	0.006	0.023	0.006	0.051	0.034
	(0.040)	(0.025)	(0.029)	(0.025)	(0.051)	(0.036)
cit	0.006*	0.002	0.004*	0.002	0.007	0.005*
	(0.003)	(0.002)	(0.002)	(0.002)	(0.003)	(0.002)
_cons	0.611**	1.413***	1.067***	1.413***	0.148	0.685**
	(0.227)	(0.162)	(0.166)	(0.162)	(0.300)	(0.218)
sigma						
_cons					1.507***	1.087***
					(0.065)	(0.047)
r2	0.192	0.201	0.210	0.201		
N	408.000	309.000	408.000	309.000	408.000	408.000
aic	1308.235	666.124	1052.793	666.124	1353.715	1136.504

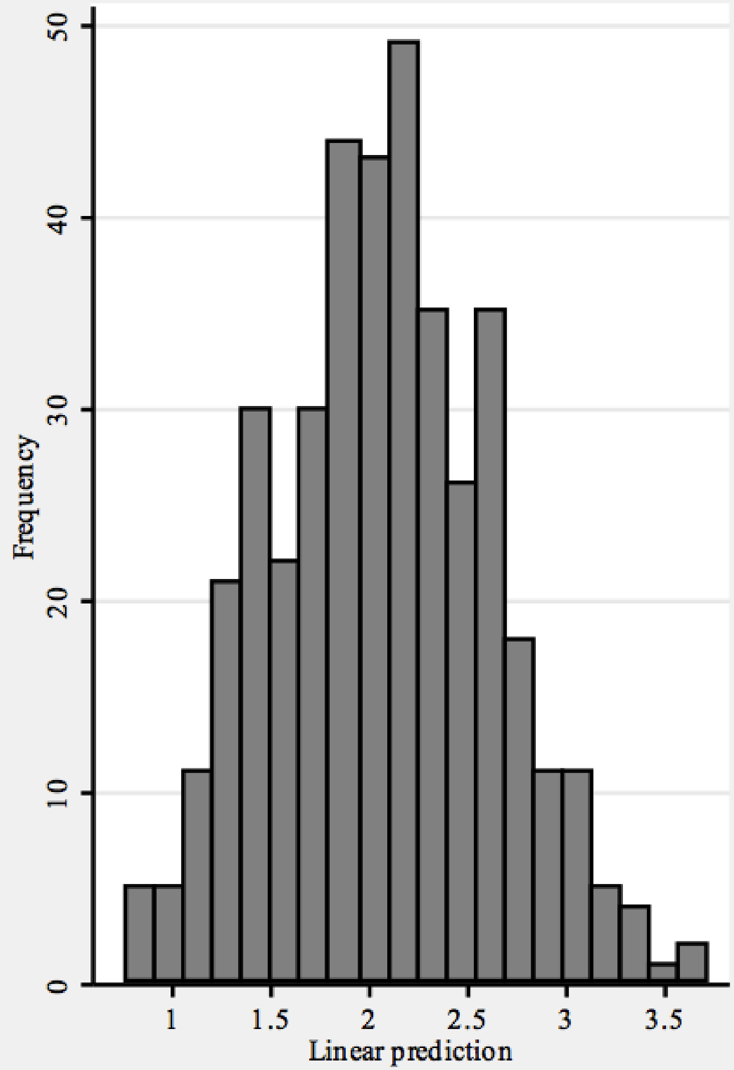
- As you can see that the coefficients are larger for *fellowship* in the tobit models.
- This is because the model takes into account that the distribution of y is censored below 1.
- Also the model is better specified with 1 as the value for a rather than 0.

Distribution of $\hat{y}$



y and *yhat*


Frequency
0
20
40
60
80
100
1
2
3
4
5
Prestige of 1st job, censored at 1


Frequency
0
10
20
30
40
50
1
1.5
2
2.5
3
3.5
Linear prediction

Interpretation: Marginal effects for left-censored

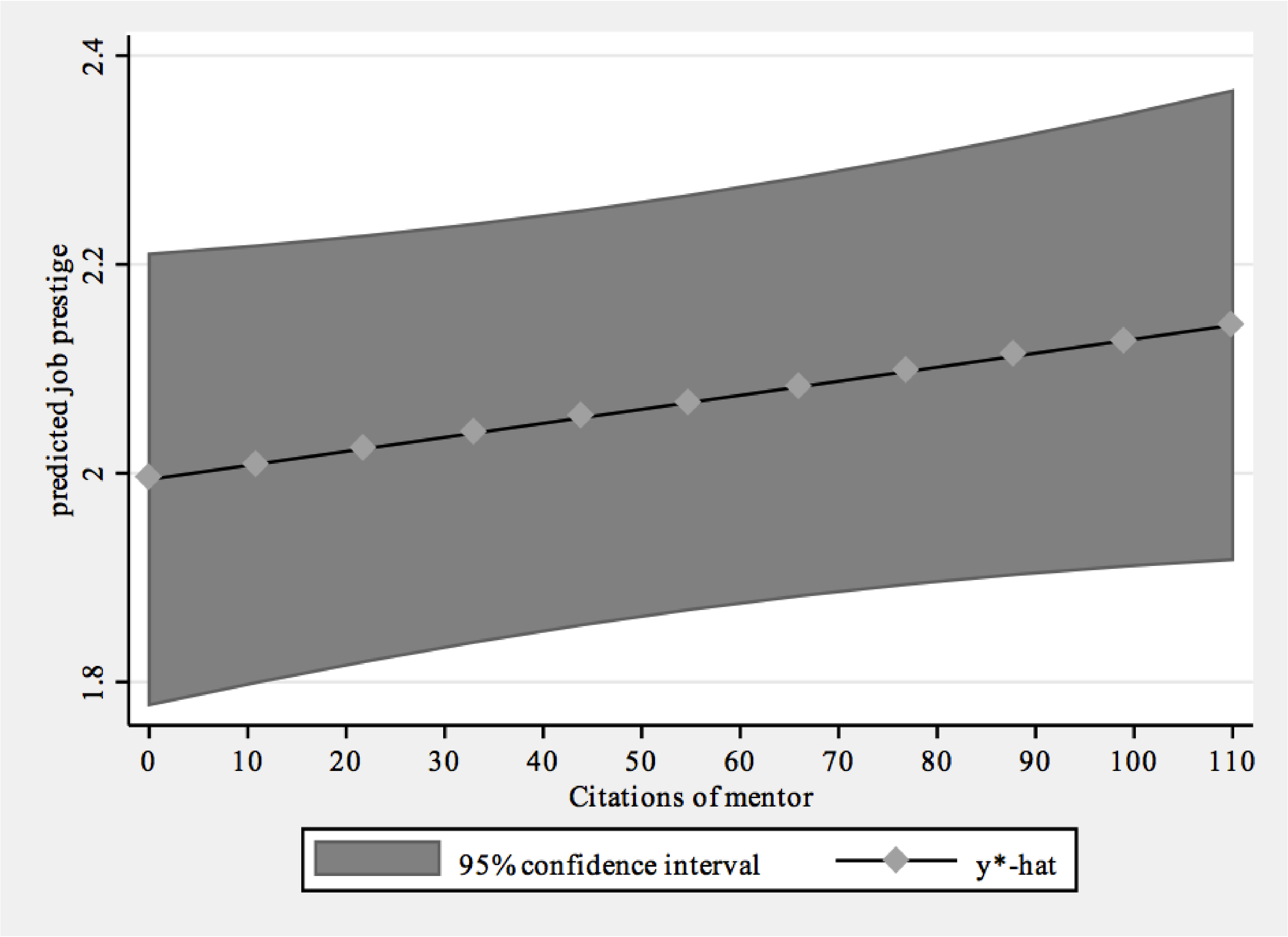
```
. margins, dydx(*) predict(e(0,.)) atmean noatlegend

Conditional marginal effects                      Number of obs   =        408
Model VCE    : OIM

Expression   : E(jobcen1|jobcen1>0), predict(e(0,.))
dy/dx w.r.t. : fem phd ment fel art cit

------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         fem |  -.2057867   .1011528    -2.03   0.042    -.4040425   -.0075309
         phd |   .2802787   .0556637     5.04   0.000     .1711799    .3893775
        ment |   .0011674   .0007713     1.51   0.130    -.0003444    .0026792
         fel |   .2826082   .1063134     2.66   0.008     .0742377    .4909787
         art |   .0294588   .0317081     0.93   0.353     -.032688    .0916055
         cit |   .0044225   .0021513     2.06   0.040     .0002059     .008639
------------------------------------------------------------------------------
```

- Marginal effects gauge the effect on the conditional mean of a change in one of the x's.

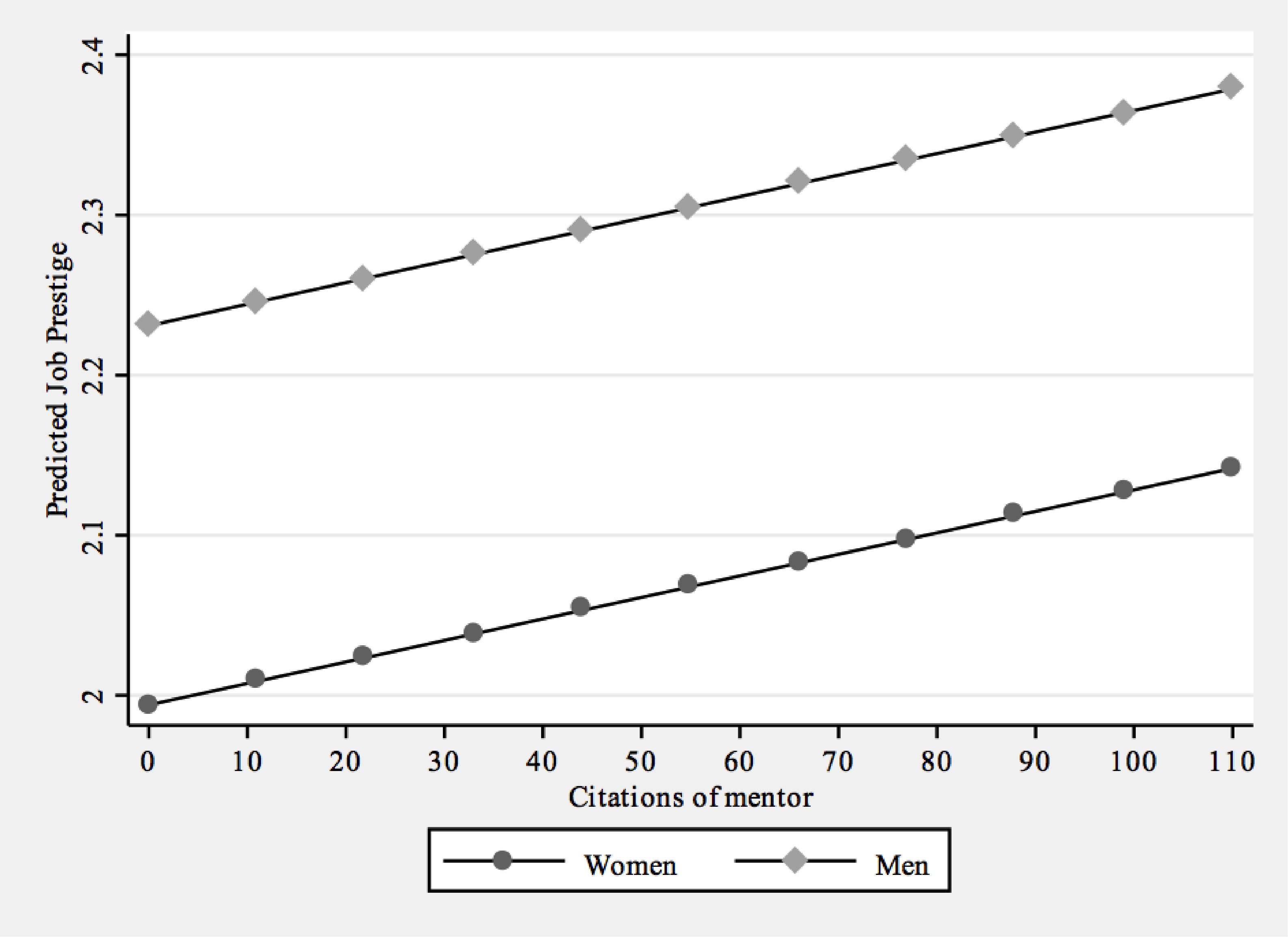
Interpretation: Marginal Effects-Without accounting for censoring

```
. margins, dydx(*) predict(ystar(0,.)) atmean noatlegend

Conditional marginal effects                      Number of obs   =        408
Model VCE    : OIM

Expression   : E(jobcen1*|jobcen1>0), predict(ystar(0,.))
dy/dx w.r.t. : fem phd ment fel art cit

------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         fem |  -.2301823   .1132266    -2.03   0.042    -.4521023   -.0082623
         phd |   .3135052    .062068     5.05   0.000     .1918542    .4351563
        ment |   .0013058   .0008625     1.51   0.130    -.0003846    .0029963
         fel |   .3161109   .1189302     2.66   0.008      .083012    .5492097
         art |    .032951   .0354692     0.93   0.353    -.0365674    .1024695
         cit |   .0049467   .0024051     2.06   0.040     .0002329    .0096606
------------------------------------------------------------------------------
```

Continuous predicted probabilities

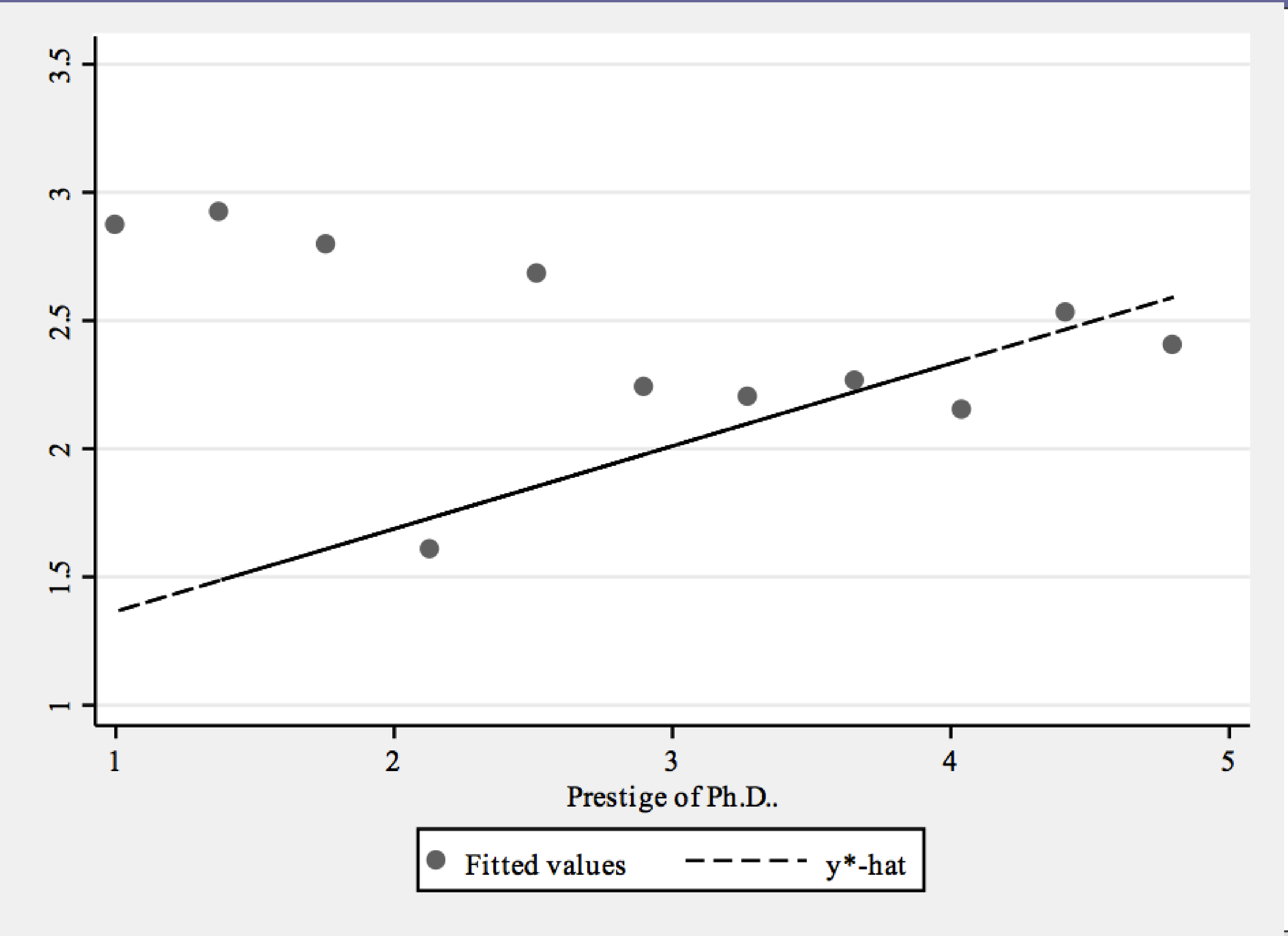


Men vs. Women



Predicted Job Prestige
2.4
2.3
2.2
2.1
2
0
10
20
30
40
50
60
70
80
90
100
110
Citations of mentor
Women
Men

Regression v. Tobit predictions



Stata code

```
prgen ment, f(0) t(110) x(fem=0 fel=1) gen(male) ci

prgen ment, f(0) t(110) x(fem=1 fel=1) gen(female) ci

** Female Grad Students **
twoway (rarea femalexblb femalexbub femalex) ///
   (connected femalexb femalex), ytitle("predicted job prestige") ///
   xtitle(Citations of mentor) xlabel (0(10)110) ///
   legend(label (1 "95% confidence interval"))

** Male Grad Students **

   twoway (rarea malexblb malexbub malex) ///
   (connected malexb malex), ytitle("predicted job prestige") ///
   xtitle(Citations of mentor) xlabel (0(10)110) ///
   legend(label (1 "95% confidence interval"))

** Both men and women **

graph twoway (connected femalexb femalex) ///
 (connected malexb malex), ytitle("Predicted Job Prestige") ///
   xtitle(Citations of mentor) xlabel (0(10)110) ///
  legend(label(1 "Women") lab(2 "Men"))
```

- What if we have both right and left censoring?
- Or if we set the cut points at different parts of the y distribution?

III. Sample Selection

- We have already touched on sample selection with our zero-inflated Poisson and negative binomial.
- These models are more prevalent in political science than the tobit model.
- This is in large part due to what we know about the data-generating processes we focus on.
- Therefore motivated by *incidental truncation*.

Incidental truncation

- Incidental truncation occurs when the observed sample arises according to values of some other unobserved or unmeasured variable and are based on the values of that variable, some observations are included, others are truncated.
- For example, democratic states might win more wars because only the most warlike democratic wars can overcome domestic opposition.

- One case that looks at incidental truncation is Reed and Clark (2000).

- There has been a growing amount of IR literature modeling the selection effects of sampling on a non-representative population.

Reed and Clark (2000)

- I thought you might be interested in an article that does not actually use any real data, but rather uses Monte Carlo simulation to model the inefficiencies that are created when you do not control for the selection of the sample into observation.

Monte Carlo Simulation

- Greene (2008) has a whole chapter (Ch. 17) looking at simulations and inference.
- Put simply, the computer generates random draws from a specified probability distribution.
- It allows you to create the population parameters, and derive distributions around those parameters.

- Reed and Clark's (2000) structural equations

$$WarOnset = \gamma - 1(Democracy) + 2(Power) + \mu$$

$$Victory = \alpha + 1(Democracy) + 2(Power) + 1.5(Initiate) + 1(Initate * Democracy) + \varepsilon$$

- Then Reed and Clark (2000) are able to vary ρ, the correlation of the errors (μ & ε) between -1 and 1 and see how the estimated parameters shift in value.

- The next step after appreciating that your sample is not necessarily randomly drawn from the population is to model this selection process.

- There are a number of different ways to do this, one of the most popular is the Heckman two-step.

Heckman selection models

- Becoming much more frequent in political science.
- Based on Heckman's (1979) two-step estimation procedure.
  - Has also been referred to as "Heckit"
- First step: estimate a probit equation
- Second step: use OLS to estimate regression on the continuous outcome.

Selection Models

- The Heckman selection model has two stages.

- In the first selection stage:

$$y_1 = \begin{cases} 1 \; if \; y^{1*} > 0 \\ 0 \; if \; y^{1*} \leq 0 \end{cases}$$

- And the second outcome stage:

$$y_2 = \begin{cases} y^{2*} \; if \; y^{1*} > 0 \\ - \quad if \; y^{1*} \leq 0 \end{cases}$$

- This means that y_2 is only observed when $y^{1*} > 0$.

- The normal specification of this model is linear in the parameters:

$$y^{1*} = \boldsymbol{\beta_1 X_1} + \varepsilon_1$$
$$y^{2*} = \boldsymbol{\beta_2 X_2} + \varepsilon_2$$

- We then maximize the following likelihood:

$$\mathrm{L}(y^* \mid y_{1i}, y_{2i}) =$$

$$\prod_{i=1}^{n} \{P(y^{1i*} \leq 0)\}^{1 - y_{1i}} \{f(y_{2i} | y^{1i*} >$$

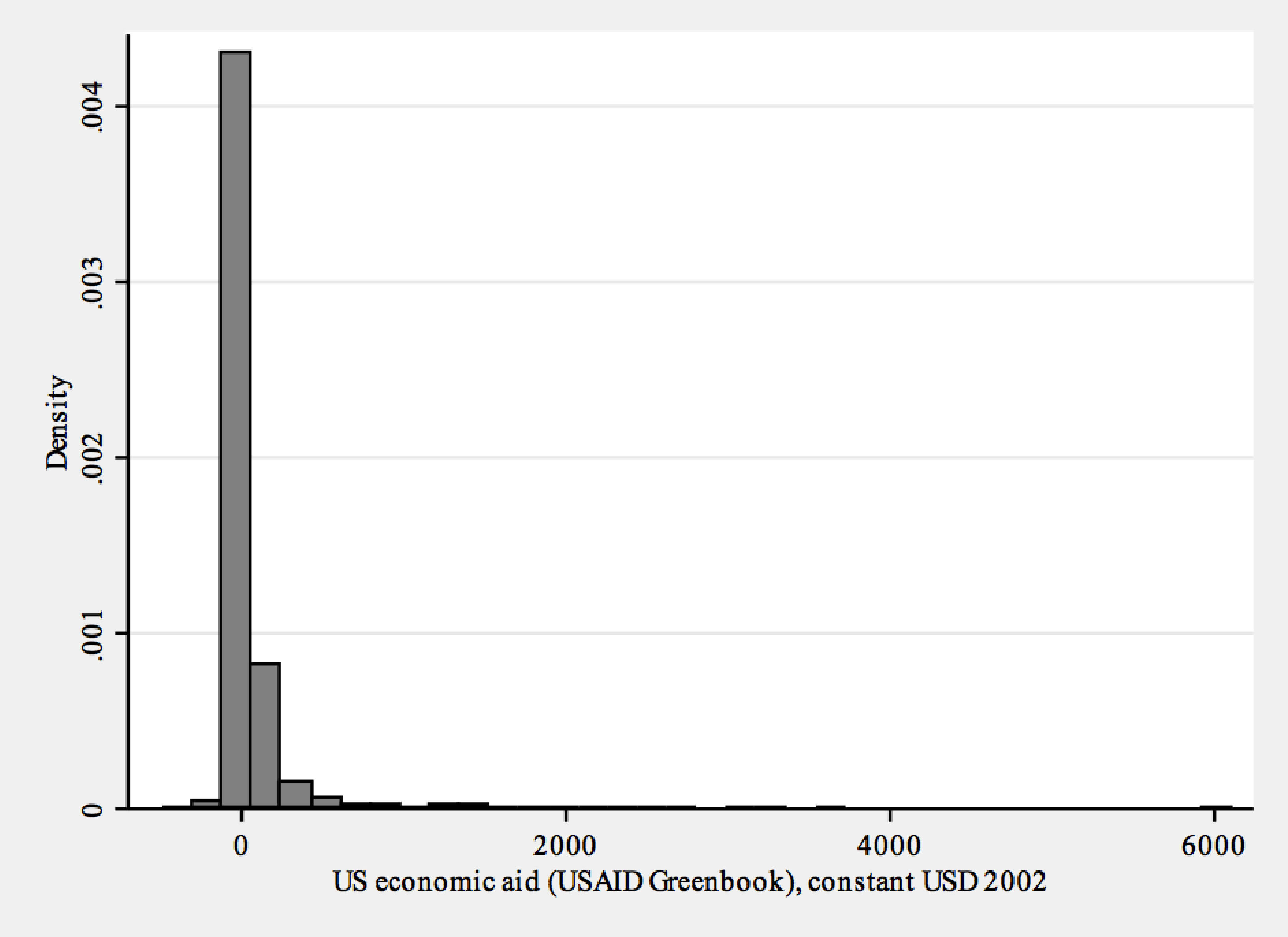
Example: US foreign aid allocation

- Is the decision to donate aid to a country related to the decision to give aid to the country at all?

- Data from Demirel-Pegg and Moskowitz (2009) article in *Journal of Peace Research*.

```
. sum EconAidConst, det

     US economic aid (USAID Greenbook), constant USD
                          2002
-------------------------------------------------------------
      Percentiles      Smallest
 1%    -43.39448      -482.7365
 5%            0      -356.1564
10%            0      -346.3981       Obs                4205
25%            0      -332.7426       Sum of Wgt.        4205

50%     12.33672                      Mean           69.33539
                        Largest       Std. Dev.      245.4955
75%     52.08845       3117.241
90%     145.0194       3261.037       Variance       60268.02
95%     272.2359       3716.031       Skewness        9.63218
99%     1324.503       6115.333       Kurtosis       145.3524
```

Many states get no aid.



Selection equation

```
passgate      |
      GDP_lg1 |  -.6306386   .1884293    -3.35   0.001    -.9999533   -.2613239
 BilTrade_lg1 |   .0202104   .0474754     0.43   0.670    -.0728398    .1132605
  polity2_lg1 |   .0338421   .0112414     3.01   0.003     .0118093    .0558749
   HR_PTS_lg2 |  -.2864174    .091479    -3.13   0.002     -.465713   -.1071219
        lnpop |  -.1417472   .0682721    -2.08   0.038    -.2755582   -.0079363
 tau_lead_lg1 |  -.0949364   .2309595    -0.41   0.681    -.5476087     .357736
 AidlessY~PCW |  -1.733909   .2768775    -6.26   0.000    -2.276579   -1.191239
   PCWspline1 |  -.2670382   .0924893    -2.89   0.004    -.4483138   -.0857625
   PCWspline2 |   .0865002   .0439979     1.97   0.049     .0002659    .1727345
   PCWspline3 |   -.000476   .0083792    -0.06   0.955     -.016899     .015947
        _cons |   9.289075   1.849895     5.02   0.000     5.663348     12.9148
```

Aid amount equation

```
 heckman lnEcAidk GDP_lg1 BilTrade_lg1 polity2_lg1 HR_PTS_lg2 lnpop if year >1990, ///
>    select (passgate = GDP_lg1 BilTrade_lg1 polity2_lg1 HR_PTS_lg2 lnpop tau_lead_lg1
AidlessYrsPCW PCWspline*) clus
> ter(ccode) nolog
Heckman selection model                          Number of obs      =      2019
(regression model with sample selection)         Censored obs       =       367
                                                 Uncensored obs     =      1652
                                                 Wald chi2(5)       =     60.08
Log pseudolikelihood = -3669.933                 Prob > chi2        =    0.0000

                                (Std. Err. adjusted for 151 clusters in ccode)
------------------------------------------------------------------------------
             |               Robust
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
lnEcAidk     |
     GDP_lg1 |  -.5929948   .1669615    -3.55   0.000    -.9202333   -.2657562
BilTrade_lg1 |   .1016733   .0738176     1.38   0.168    -.0430065     .246353
 polity2_lg1 |   .0418295   .0190612     2.19   0.028     .0044703    .0791888
  HR_PTS_lg2 |  -.3021418   .1174158    -2.57   0.010    -.5322725   -.0720112
       lnpop |   .1850348   .1114746     1.66   0.097    -.0334514    .4035211
       _cons |   6.245198   1.643951     3.80   0.000     3.023112    9.467283
```

```
 heckman lnEcAidk GDP_lg1 BilTrade_lg1 polity2_lg1 HR_PTS_lg2 lnpop if year >1990, ///
>    select (passgate = GDP_lg1 BilTrade_lg1 polity2_lg1 HR_PTS_lg2 lnpop tau_lead_lg1 AidlessYrsPCW PCWspline*) clus
> ter(ccode) nolog
Heckman selection model                         Number of obs      =      2019
(regression model with sample selection)        Censored obs       =       367
                                                Uncensored obs     =      1652
                                                Wald chi2(5)       =     60.08
Log pseudolikelihood = -3669.933                Prob > chi2        =    0.0000

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------------------------------------------------------------------------------
             |               Robust
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       _cons |   6.245198   1.643951     3.80   0.000     3.023112    9.467283
-------------+----------------------------------------------------------------
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  HR_PTS_lg2 |  -.2864174    .091479    -3.13   0.002     -.465713   -.1071219
       lnpop |  -.1417472   .0682721    -2.08   0.038    -.2755582   -.0079363
tau_lead_lg1 |  -.0949364   .2309595    -0.41   0.681    -.5476087     .357736
AidlessY~PCW |  -1.733909   .2768775    -6.26   0.000    -2.276579   -1.191239
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  PCWspline2 |   .0865002   .0439979     1.97   0.049     .0002659    .1727345
  PCWspline3 |   -.000476   .0083792    -0.06   0.955     -.016899     .015947
       _cons |   9.289075   1.849895     5.02   0.000     5.663348     12.9148
-------------+----------------------------------------------------------------
     /athrho |    -.65379   .1437992    -4.55   0.000    -.9356313   -.3719487
    /lnsigma |   .6183054    .055078    11.23   0.000     .5103545    .7262562
-------------+----------------------------------------------------------------
         rho |  -.5742159   .0963852                     -.7332086    -.355695
       sigma |   1.855781   .1022126                      1.665882    2.067326
      lambda |  -1.065619   .2066768                     -1.470698   -.6605396
------------------------------------------------------------------------------
Wald test of indep. eqns. (rho = 0): chi2(1) =    20.67   Prob > chi2 = 0.0000
```

- Stata conducts a likelihood ratio test to see if the errors of the two equations are correlated.

- The chi^2 with 1 degree of freedom equals 20.67, which allows us to reject the null that the decision to give any aid and how much aid are not related.

Heckman extensions

- This technique has also been used to model both a selection and an outcome variable that is dichotomous.

Multiple equation models

- This is now getting into the territory of the multiple equation models, which we will see next week.

- Questions?